CSC108: Introduction to Computer Programming

Lecture 11

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Recursion

Recursive Definitions

- A description of something that refers to itself is called a *recursive* definition.

- Have you had a teacher tell you that you can't use a word in its own definition? This is a *circular* definition.

- In mathematics, recursion is frequently used. The most common example is the factorial:

  For example, $5! = 5(4)(3)(2)(1)$,

  or

  $5! = 5(4!)$

Recursive Definitions

- In other words, $n! = n(n-1)!$

- Or

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n(n-1)! & \text{otherwise} \end{cases}$$

- This definition says that 0! is 1, while the factorial of any other number is that number times the factorial of one less than that number.

Recursive Definitions

- Our definition is recursive, but definitely <u>not</u> circular. Consider 4!
  - 4! = 4(4-1)! = 4(3!)
  - What is 3!? We apply the definition again
    4! = 4(3!) = 4[3(3-1)!] = 4(3)(2!)
  - And so on…
    4! = 4(3!) = 4(3)(2!)
    = 4(3)(2)(1!)
    = 4(3)(2)(1)(0!)
    = 4(3)(2)(1)(1)
    = 24

Recursive Definitions

- Factorial is not circular because we eventually get to 0!, whose definition does not rely on the definition of factorial and is just 1. This is called a *base case* for the recursion.

- When the base case is encountered, we get a closed expression that can be directly computed.

Recursive Definitions

- All good recursive definitions have these two key characteristics:
  - There are one or more base cases for which no recursion is applied.
  - One or more recursive case which eventually end up at one of the base cases.

- The simplest way for these two conditions to occur is for each recursion to act on a smaller version of the original problem. A very small version of the original problem that can be solved without recursion becomes the base case.

Recursive Definitions

- We've seen previously that factorial can be calculated using a loop accumulator.

- If factorial is written as a separate function:

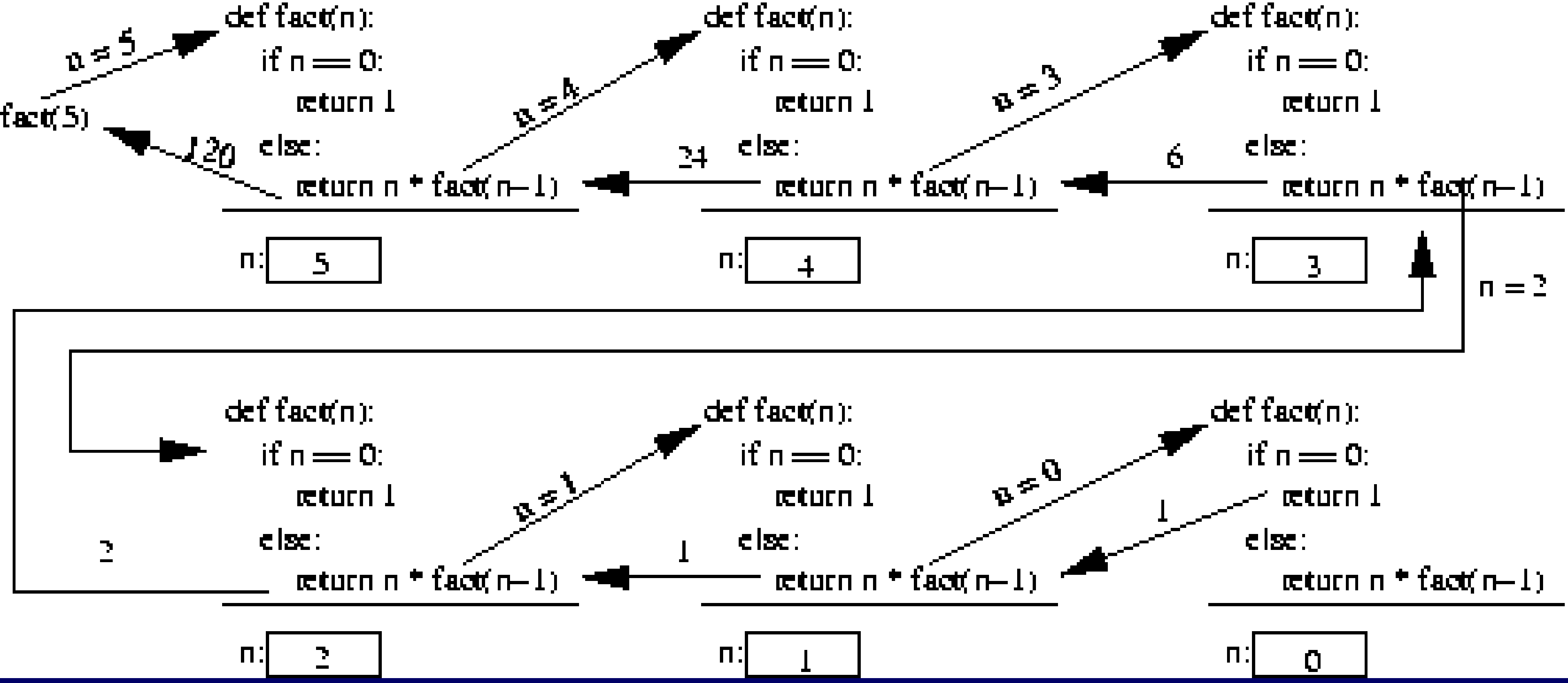
```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```

Recursive Definitions

- We've written a function that calls *itself*, a *recursive function*.

- The function first checks to see if we're at the base case (`n==0`). If so, return 1. Otherwise, return the result of multiplying *n* by the factorial of *n-1*, `fact(n-1)`.

Recursive Definitions



Functions
(revisited)

First-Class Objects

- The term '**first-class object' refers to an object** which has the following important properties:

  - can be stored in variables and data structures

  - can be passed to and returned by a function

- In all programming languages, primitive data types (ints, floats, strings, etc.) are first-class since they conform to the above rules.

- In many of those, functions are not first-class. However, in Python functions are in fact first-class.

- What does this entail?

Representing Functions

- We've alluded to the fact that Python stores function names the same way it stores variable names.
- In a namespace, all Python really has is names that are
- connected to memory addresses (also called pointers).
- For immutable data, those addresses point to primitive data objects like 4. For mutable data, they point to more complex objects, which may have pointers of their own
- For functions, addresses point to the memory space where the low-level commands of the function are stored.

Functions as Variables

- Moreover, it turns out that functions are in fact objects, meaning that they can be manipulated as such.

- For instance we can assign function names to other variables:

```
def even(i):
    return i % 2 == 0

print even(3)
not_odd = even
print not_odd(3)
```

Functions as Parameters

- We can pass function names to other functions as parameters:

```
def add(i,j):
        return i + j
def multiply(i,j):
        return i * j
add(3,5)                # returns 8
multiply(3,5)           # returns 15
def do(fxn, a, b):
        return fxn(a,b)

do(add,3,5)             # calls add, returns 8
```

Functions as First-Class Objects

- This is a varsity-level feature of Python that few programming languages share.
- Using functions as first-class objects (passing them around, renaming them, etc.) affords us some great flexibility when writing code.
- For instance, let's write a function that times the execution of a single-argument function and returns the time it took in seconds.

A Function Timer

```
import time
def runtime(f, arg):
'''f is a 1-argument function. arg is a suitable argument for f. Return the amount of time it takes to run f on arg.'''
    before = time.clock()
    f(arg)
    after = time.clock()
    return (after - before)
```

Using the Timer

- For example, if we wanted to time this function to find which is faster of bubble or selection sort

- We could use:

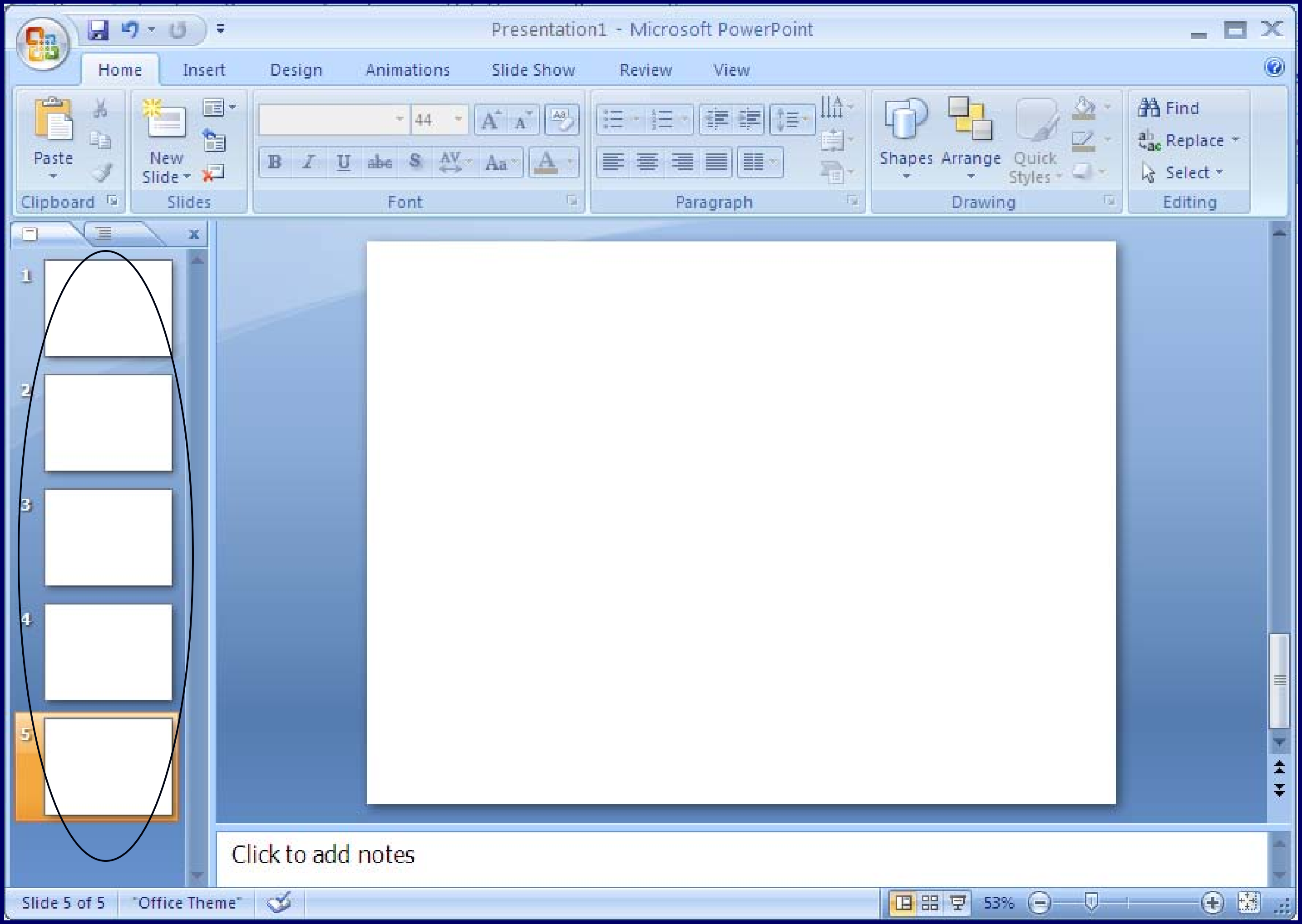
```
print "%1.20f" % runtime(bubble_sort,[3,6,5,4,1])
print "%1.20f" % runtime(selection_sort,[3,6,5,4,1])
```

Data Structures (revisited)

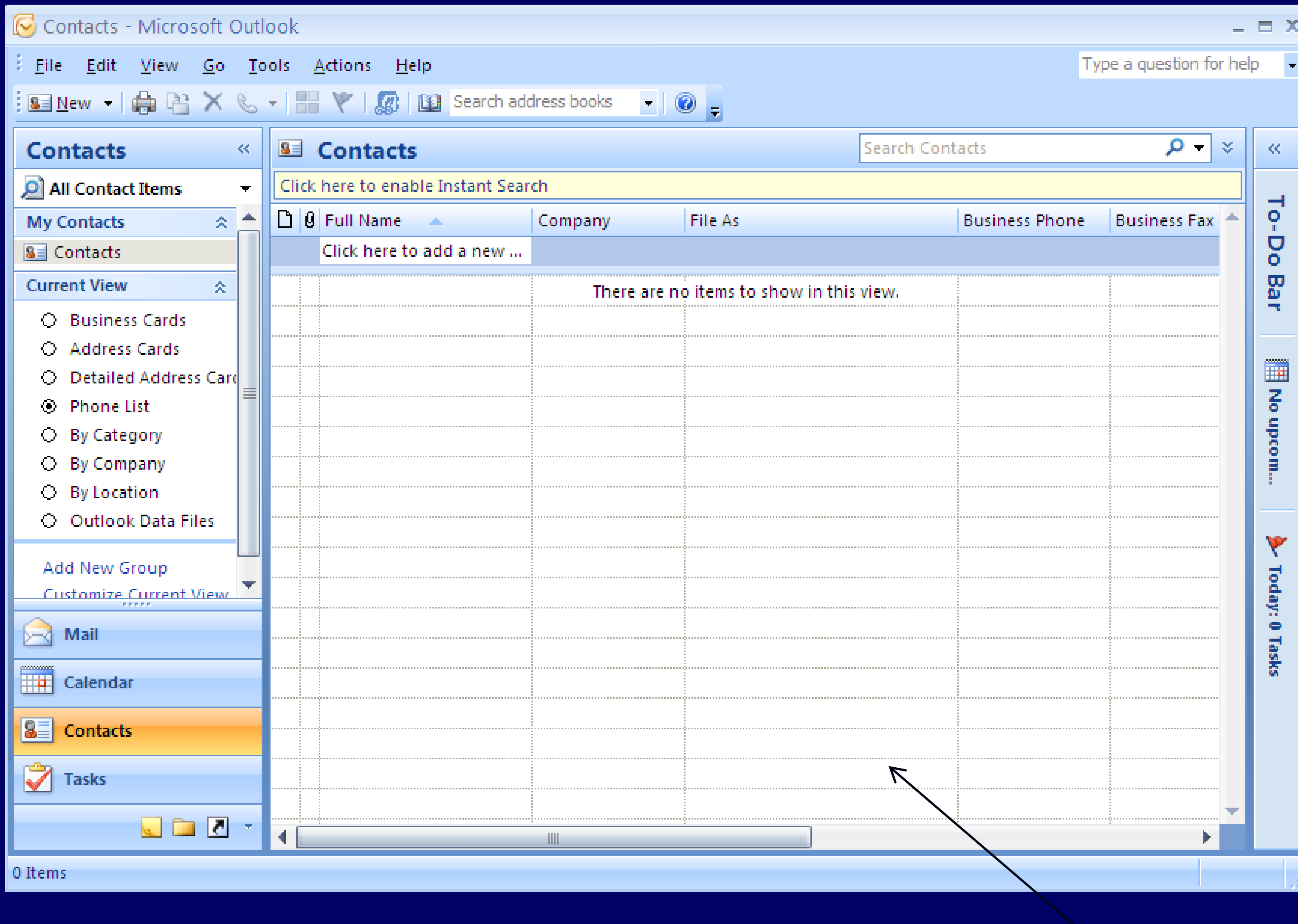
Data structures

- We have learnt the following data structures:
  - List
  - Tuple
  - Dictionary
- The Type of the data structure you use is dependent on the functionality required.

Which data structure should be used here?



Which data structure should be used here?



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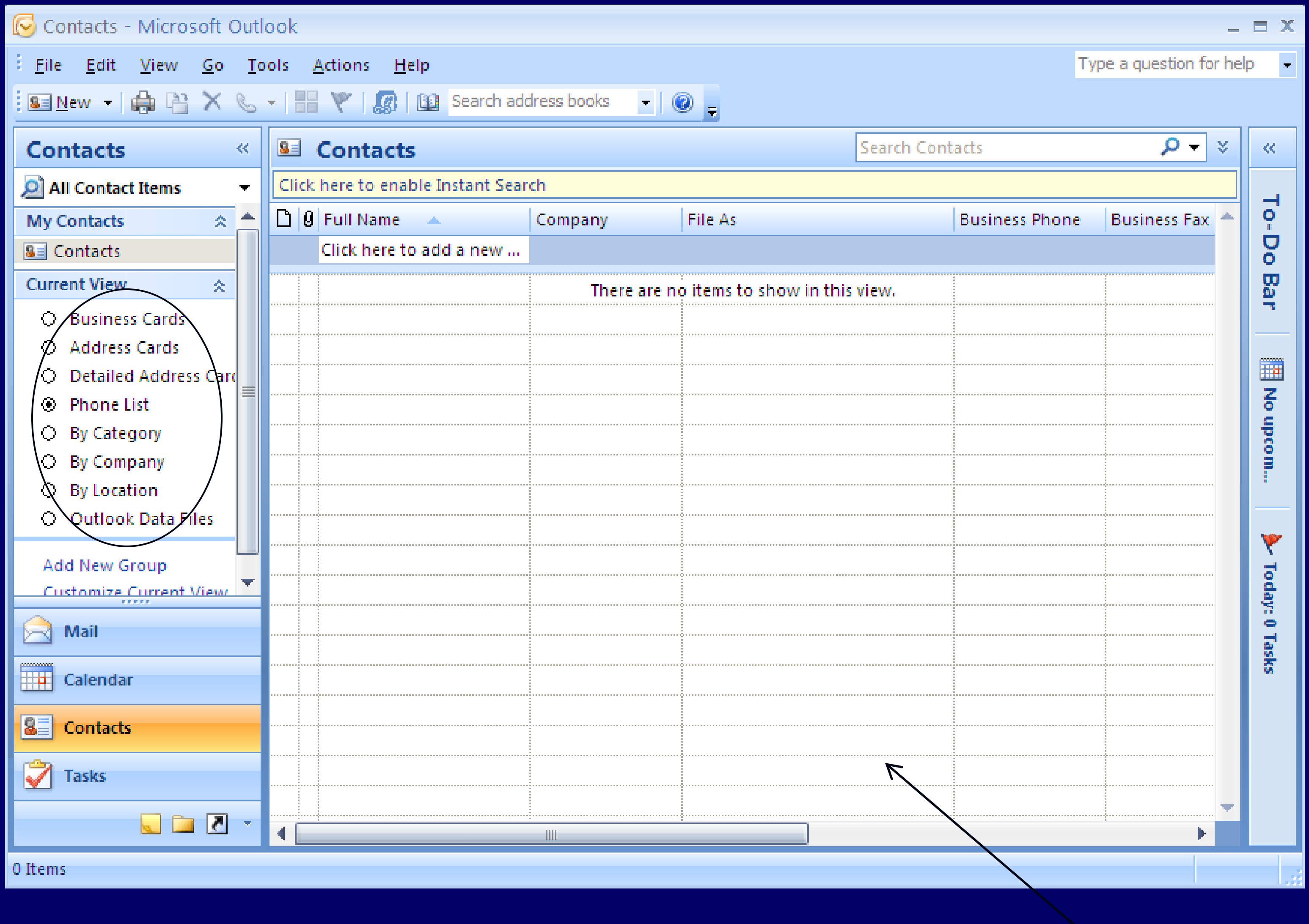

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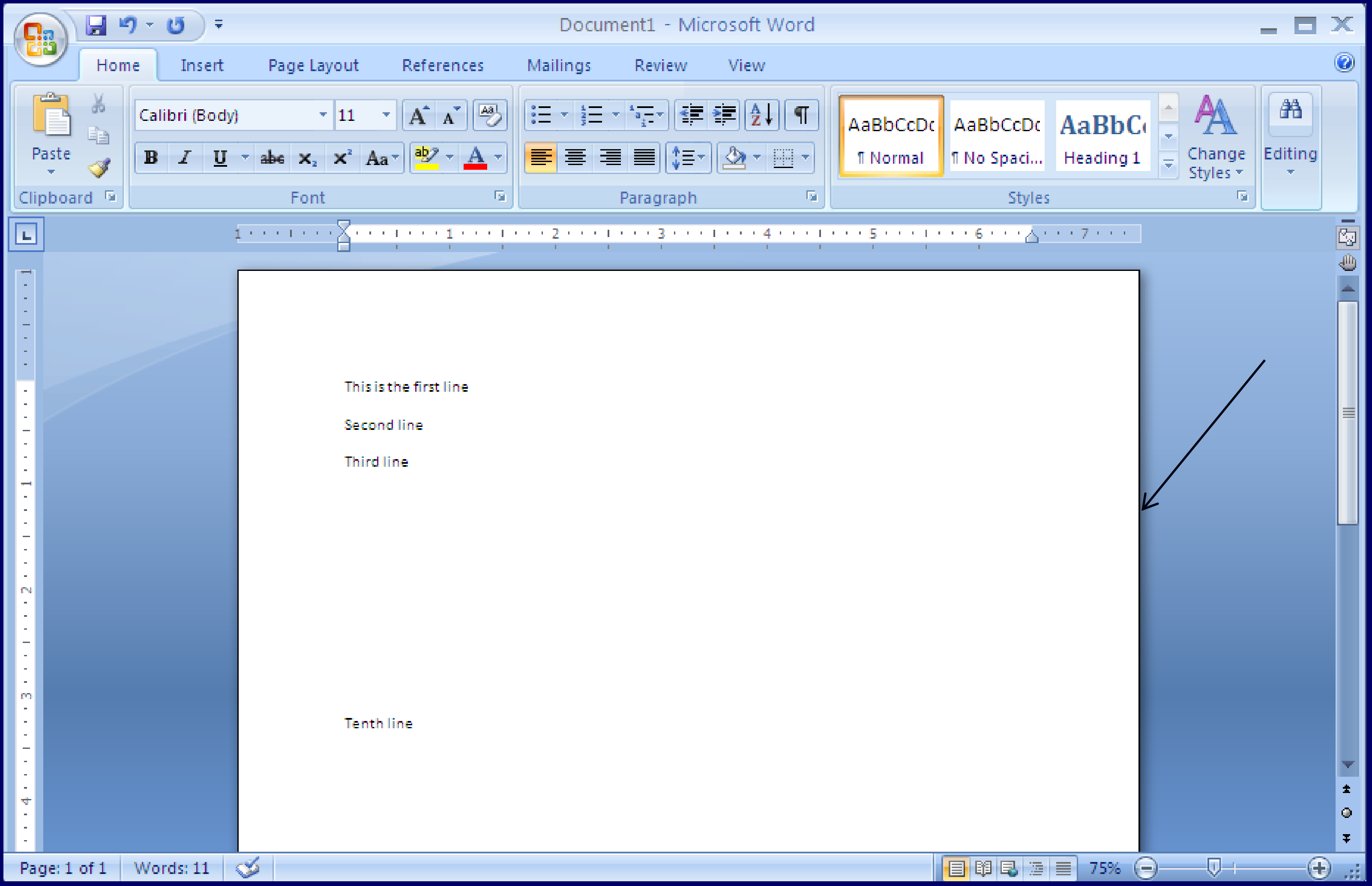

This Week’s To Do List

- Go through lecture slides – make sure you try the code snippets
- Try the lecture’s programs posted on course website