



CSCD43: Database Systems Technology

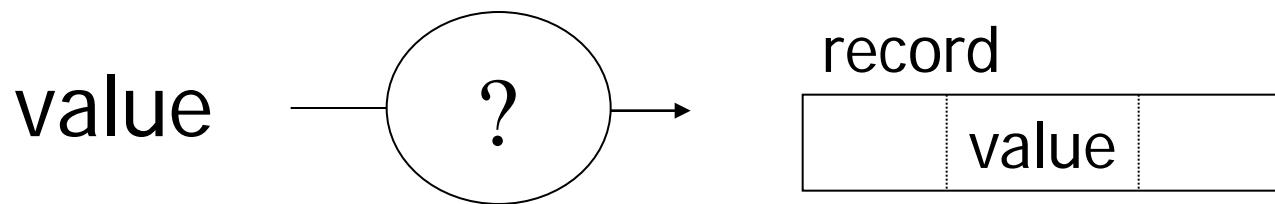
Lecture 7

Wael Aboulsaadat

Acknowledgment: these slides are based on Prof. Garcia-Molina & Prof. Ullman slides accompanying the course's textbook.



CSCD43: Database Systems Technology



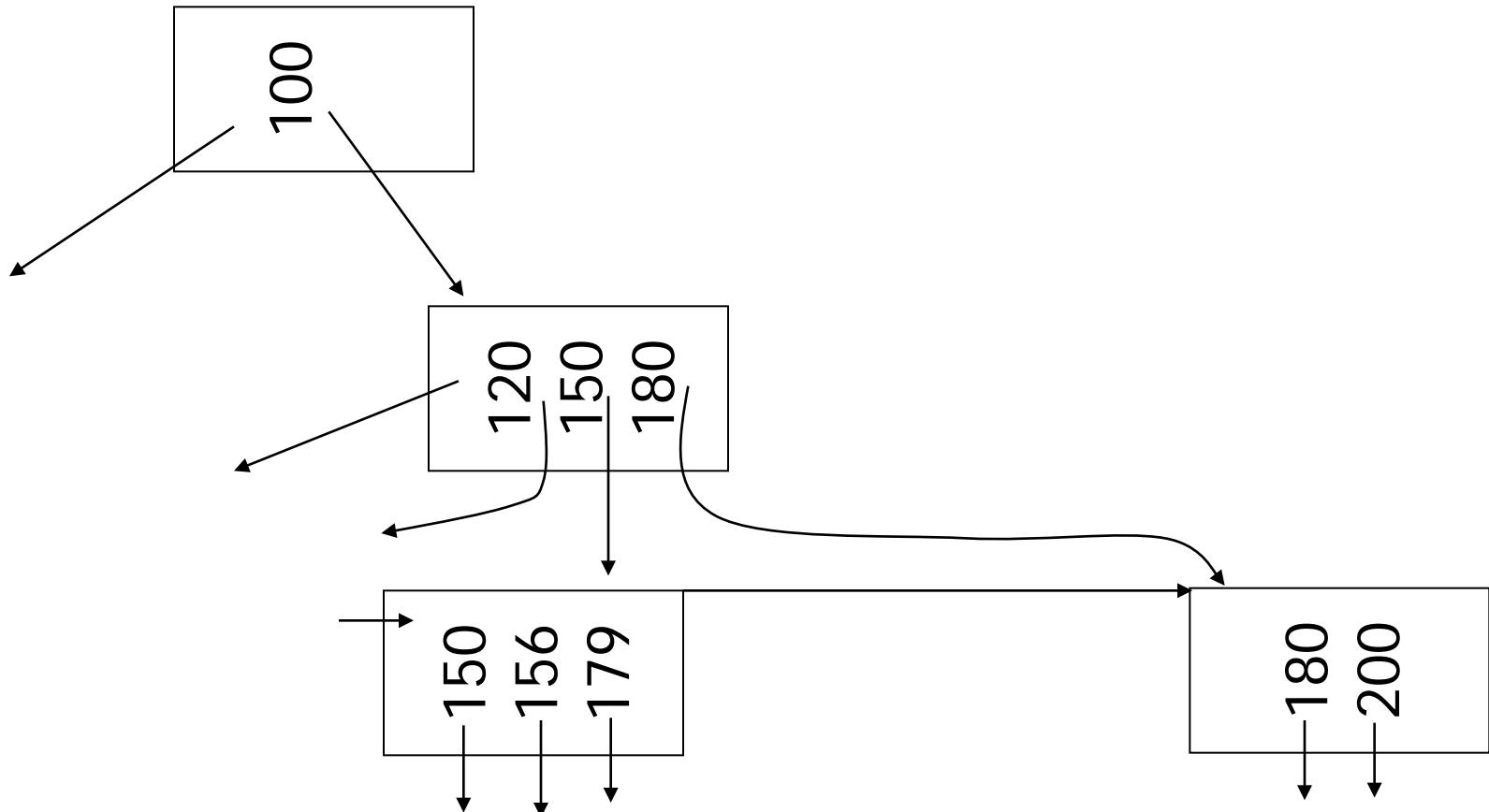


Topics

- Conventional Indexes
 - B-trees
- Hashing Schemes
- Bitmap Indexes

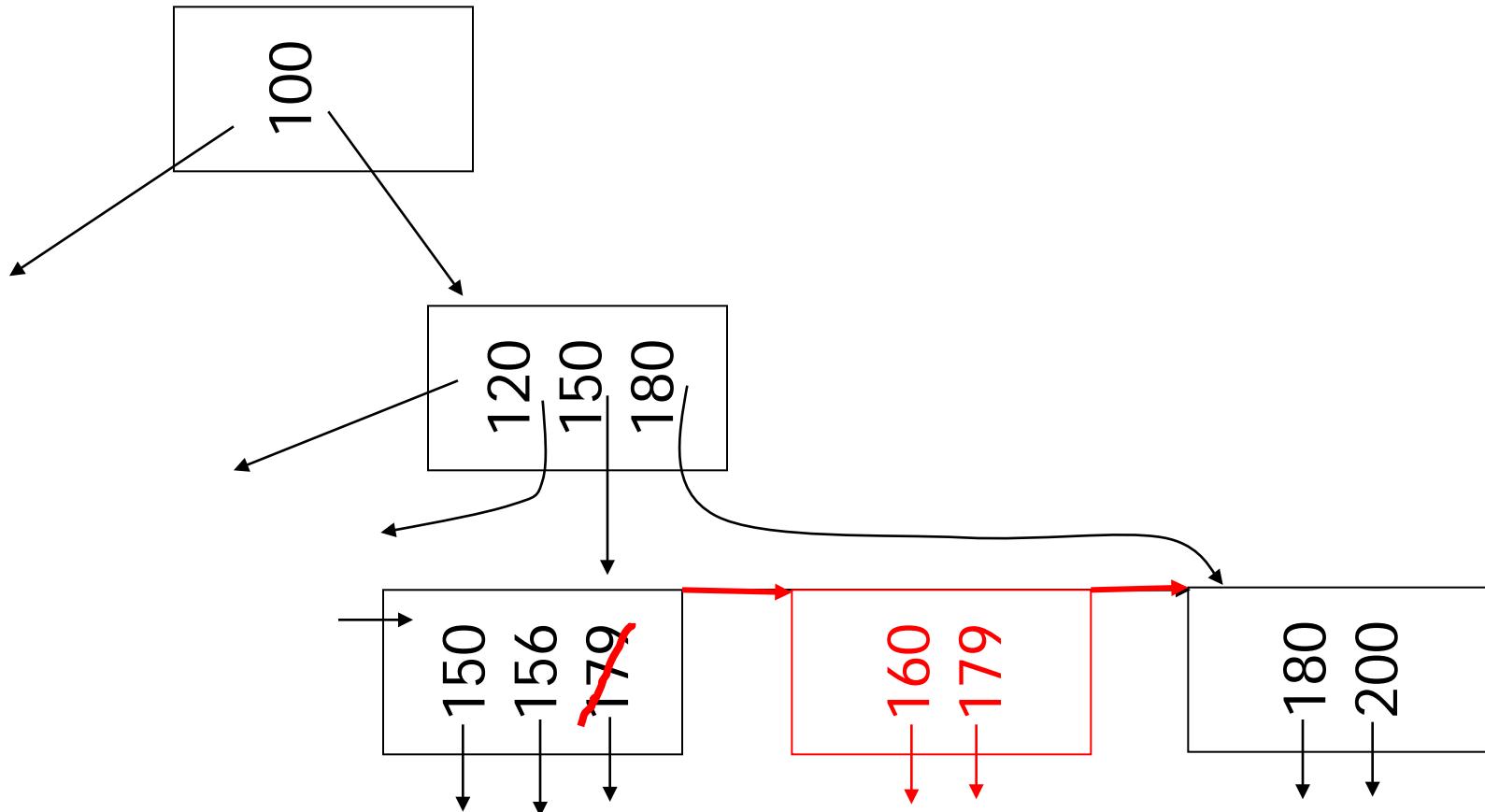
(c) Insert key = 160

n=3

 $(n+1)/2$ 

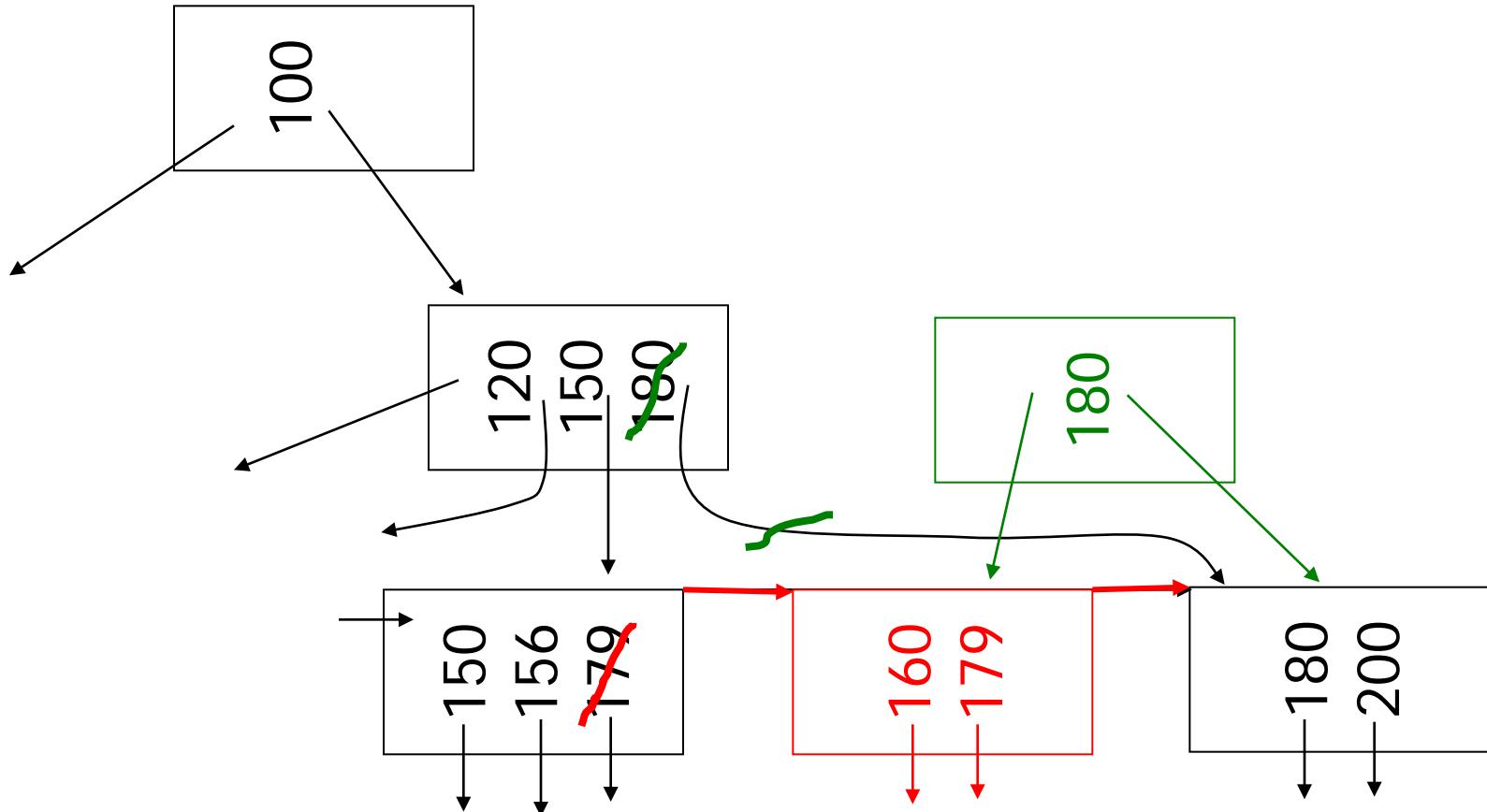
(c) Insert key = 160

n=3

 $(n+1)/2$ 

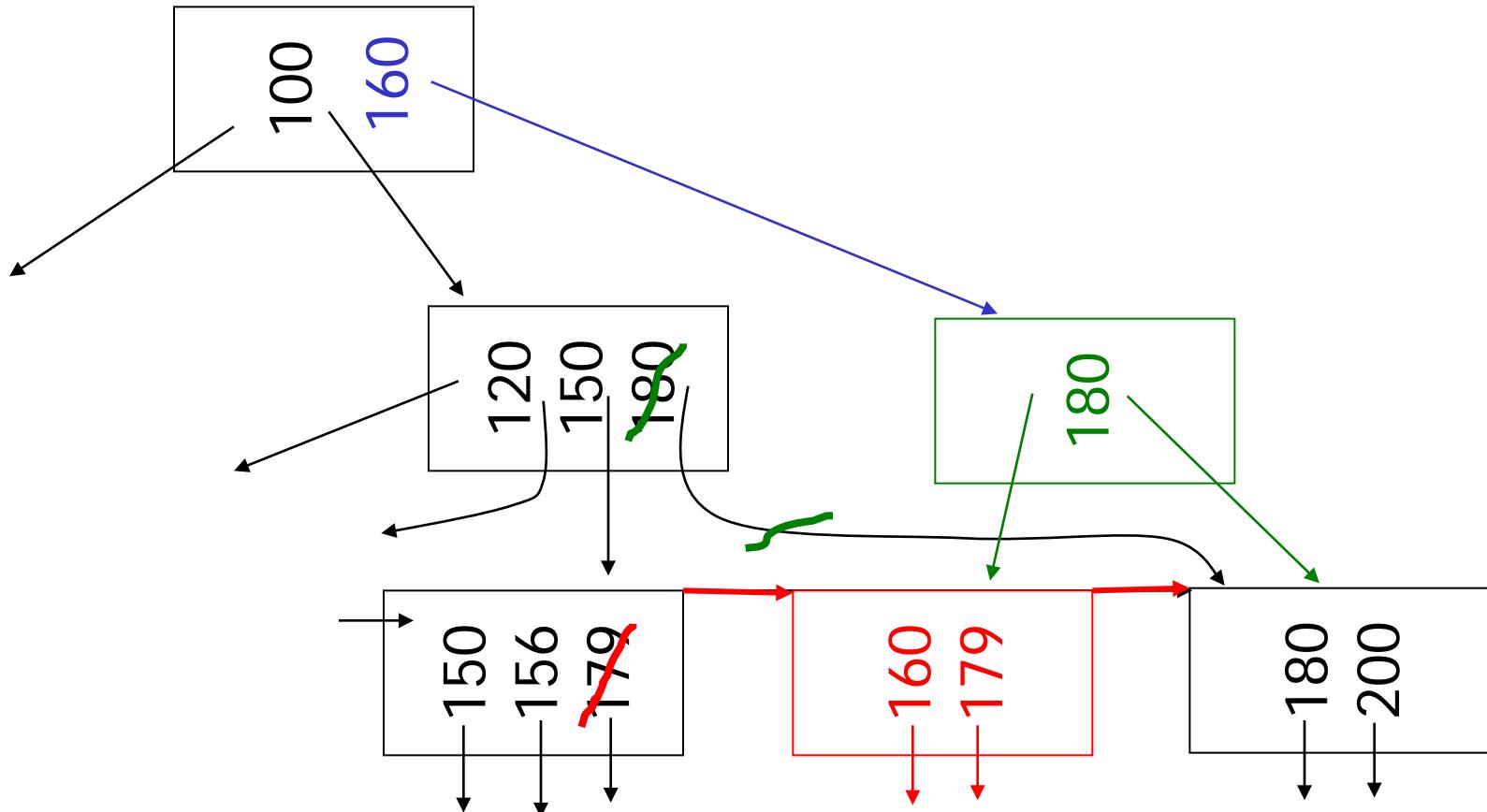
(c) Insert key = 160

n=3

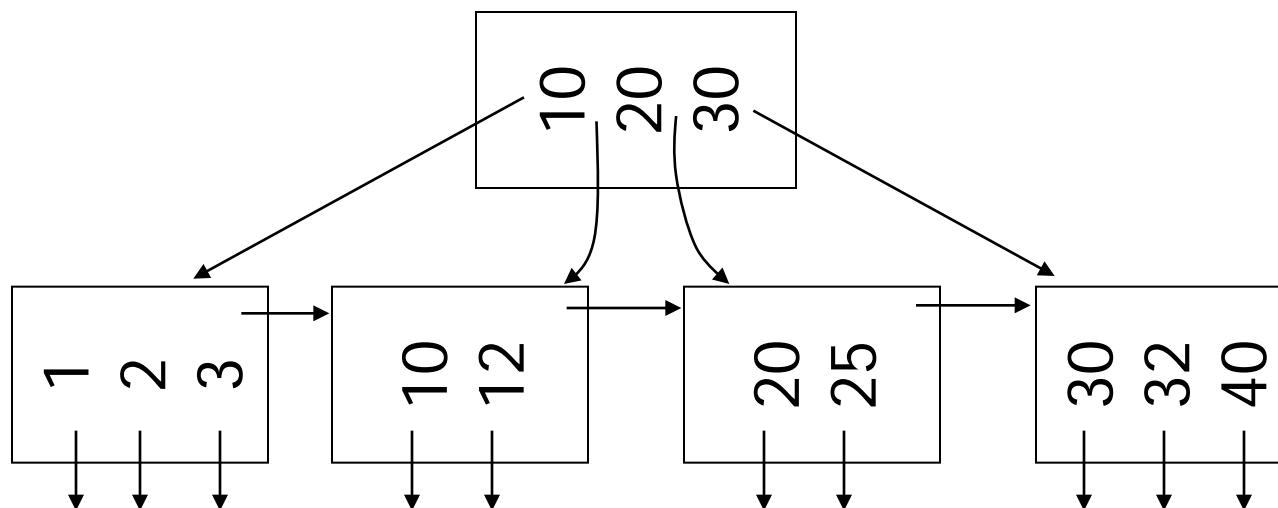
 $(n+1)/2$ 

(c) Insert key = 160

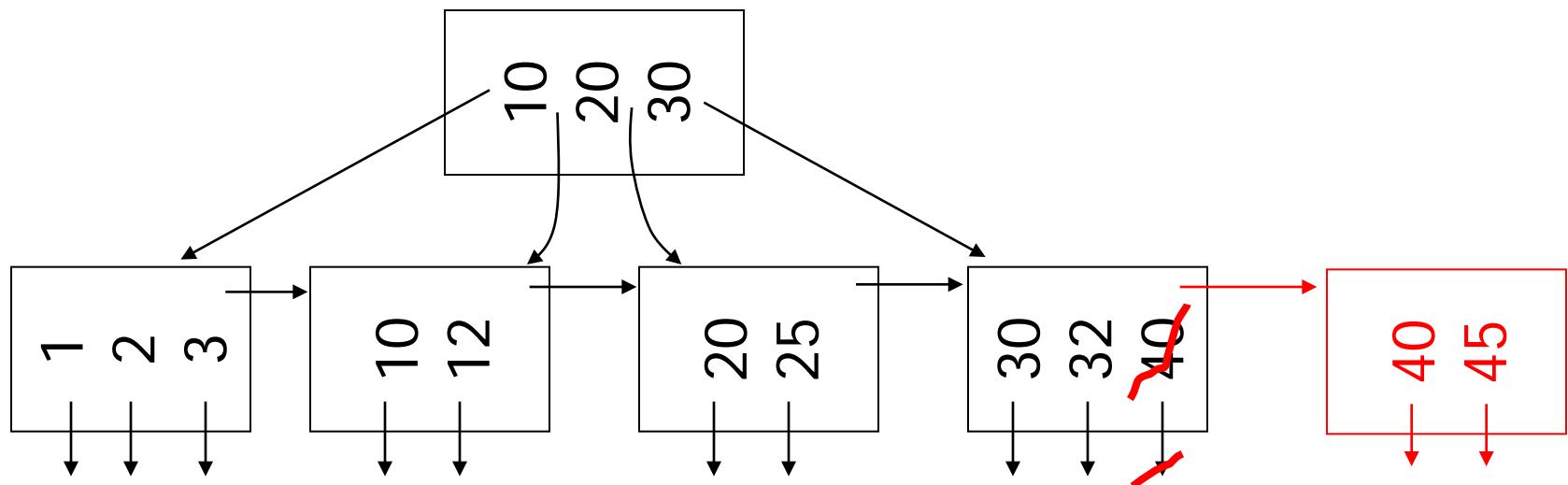
n=3

 $(n+1)/2$ 

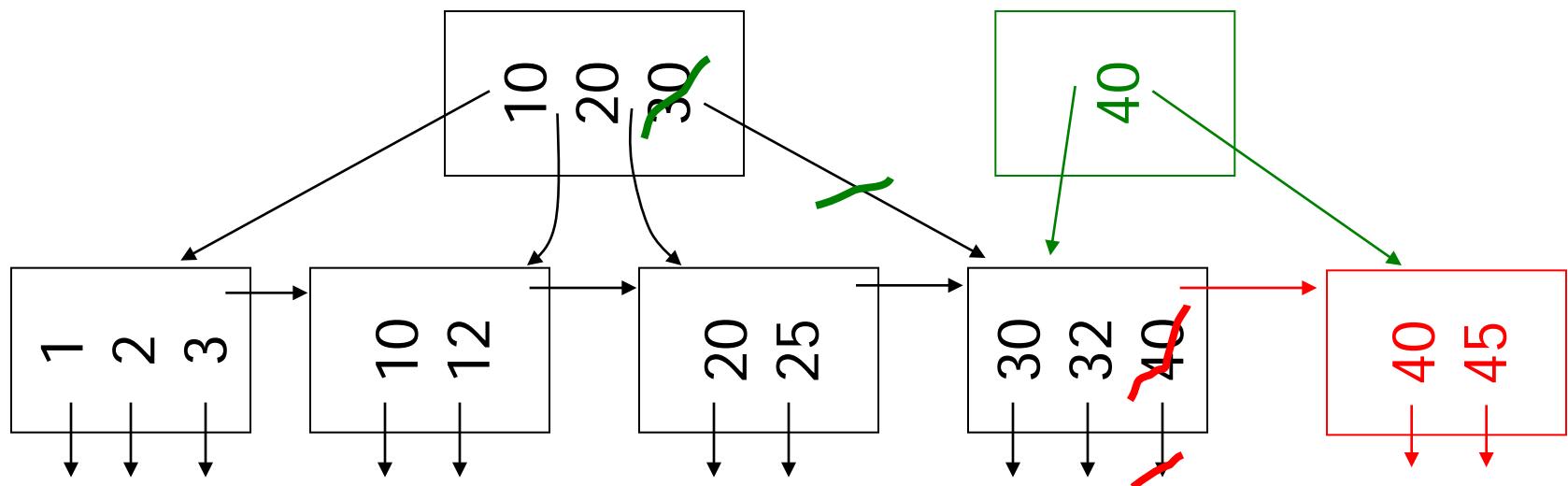
(d) New root, insert 45

 $n=3$ $(n+1)/2$ 

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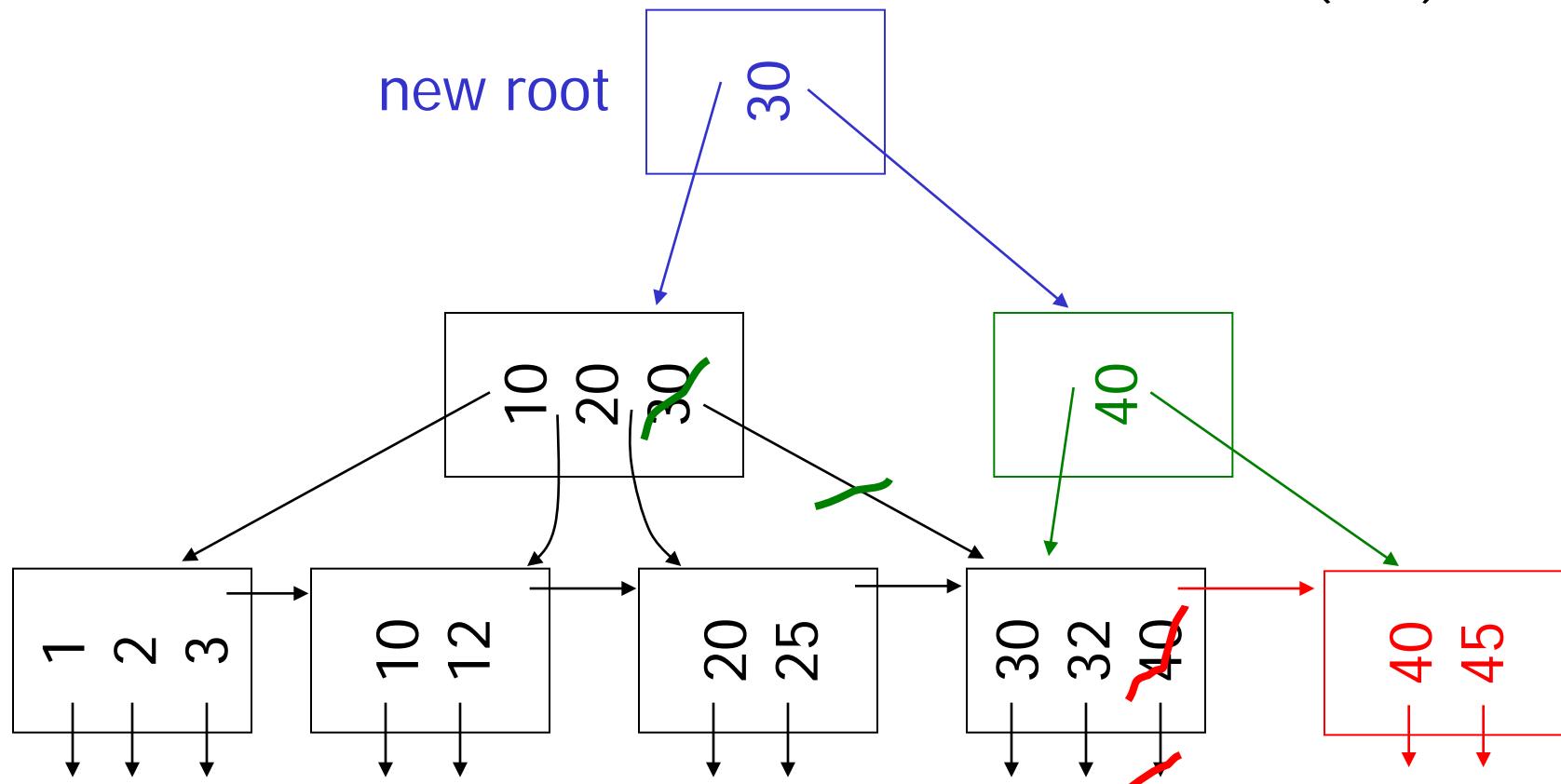
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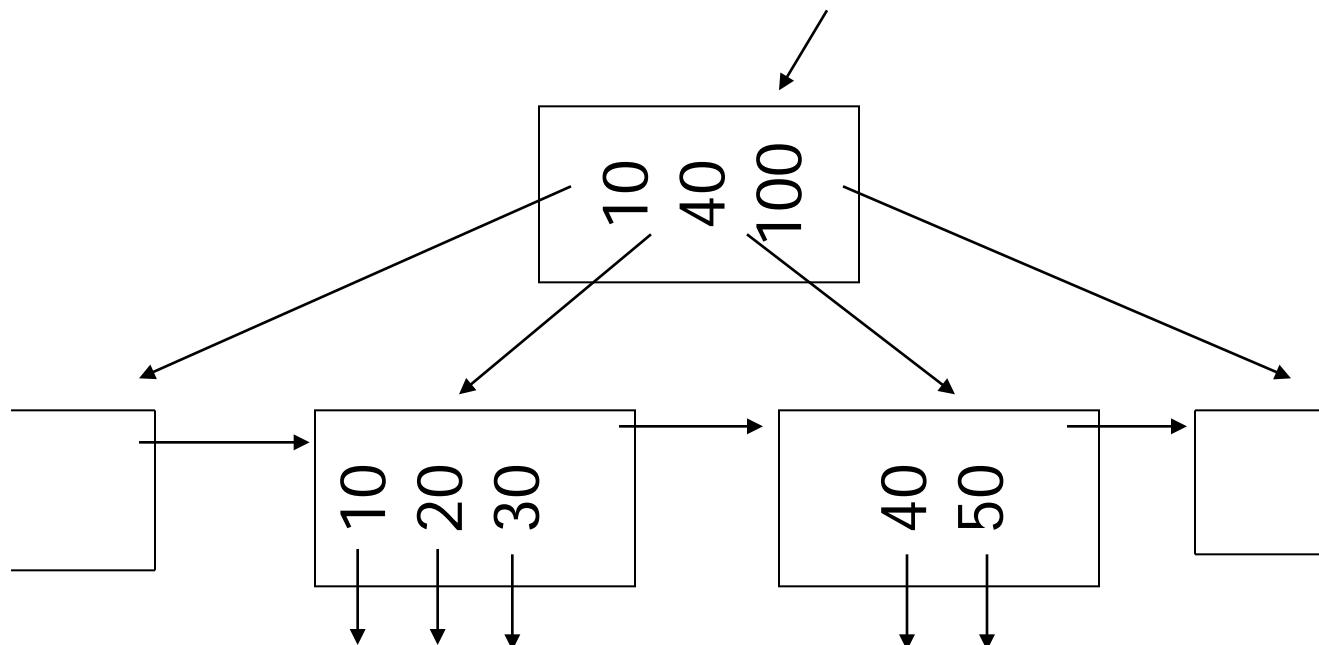


Deletion from B+tree

- (a) Simple case - no example
- (b) Coalesce with neighbor (sibling)
- (c) Re-distribute keys
- (d) Cases (b) or (c) at non-leaf

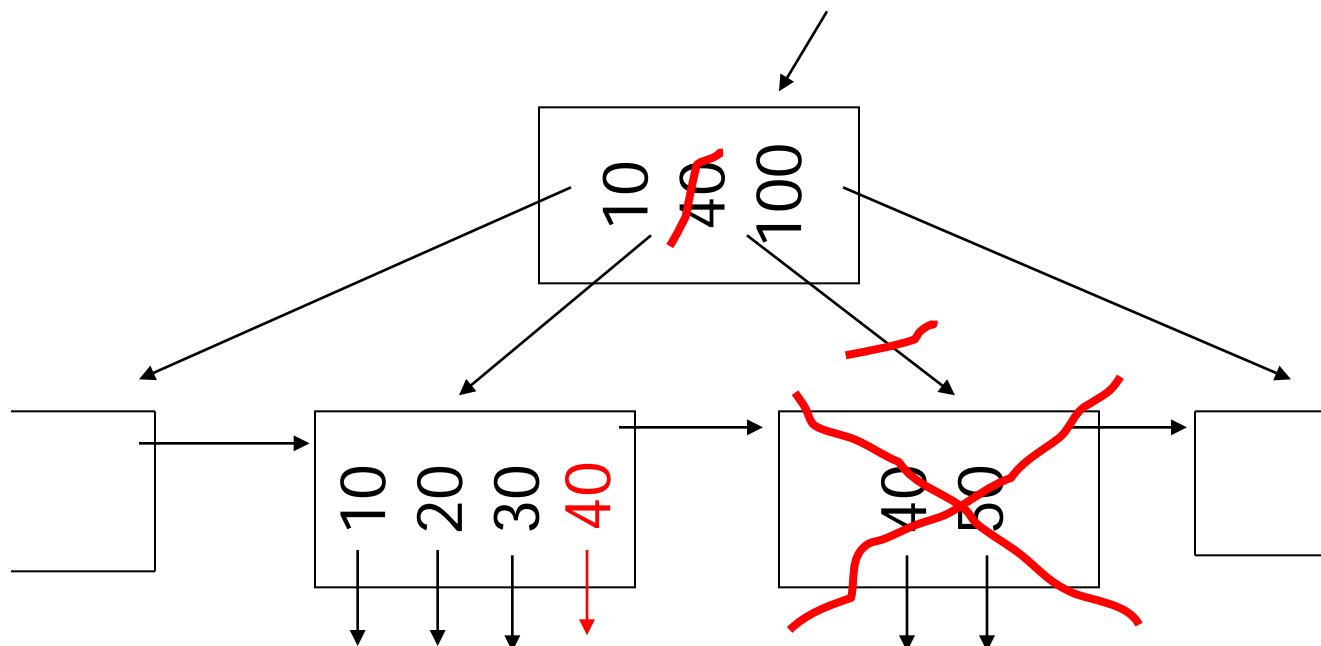
(b) Coalesce with sibling
– Delete 50

$$n=4$$
$$(n+1)/2$$



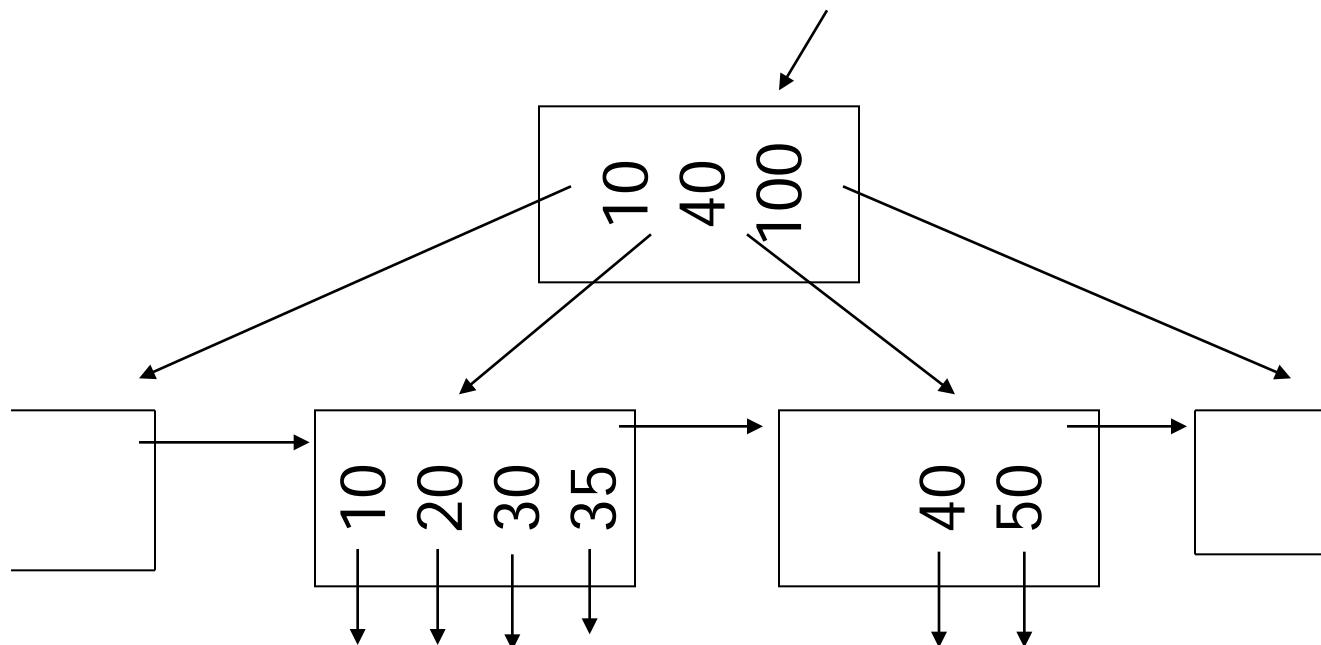
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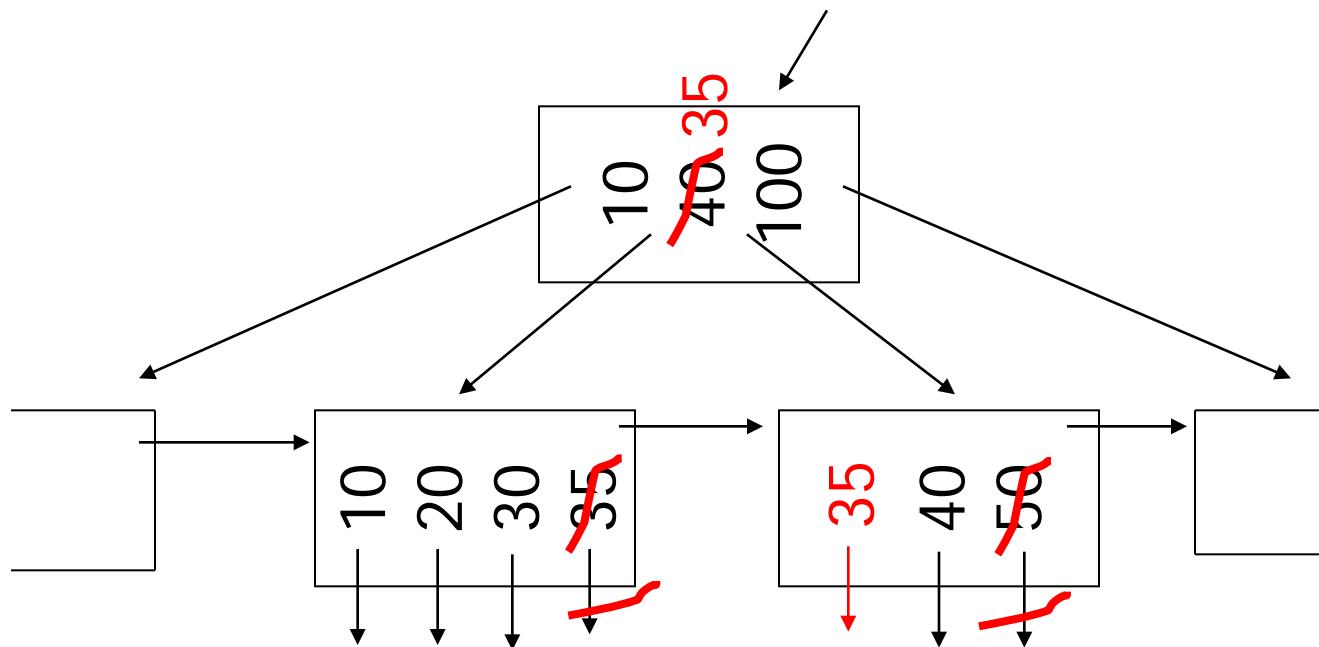
(c) Redistribute keys – Delete 50

$n=4$
 $(n+1)/2$



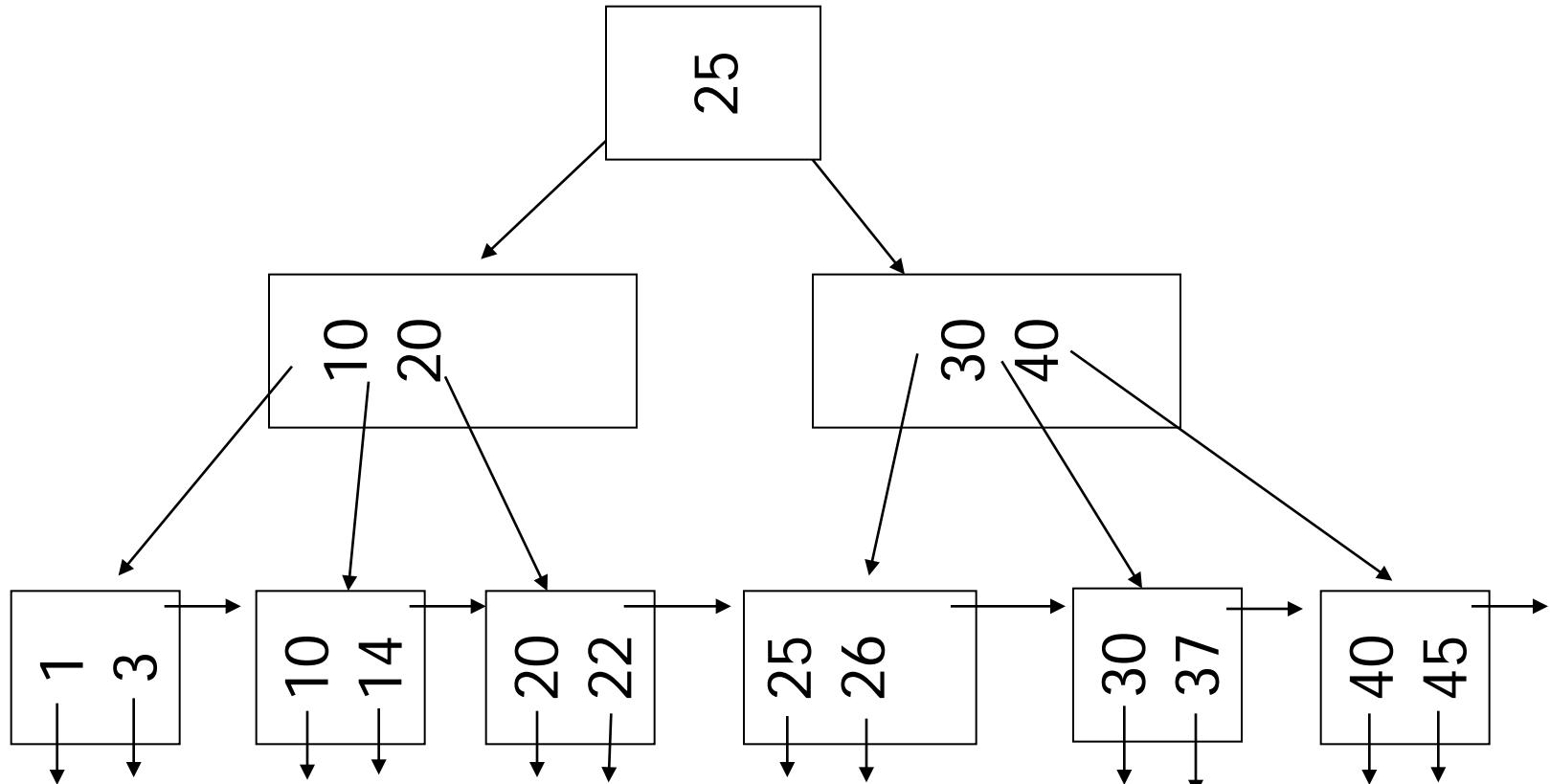
(c) Redistribute keys – Delete 50

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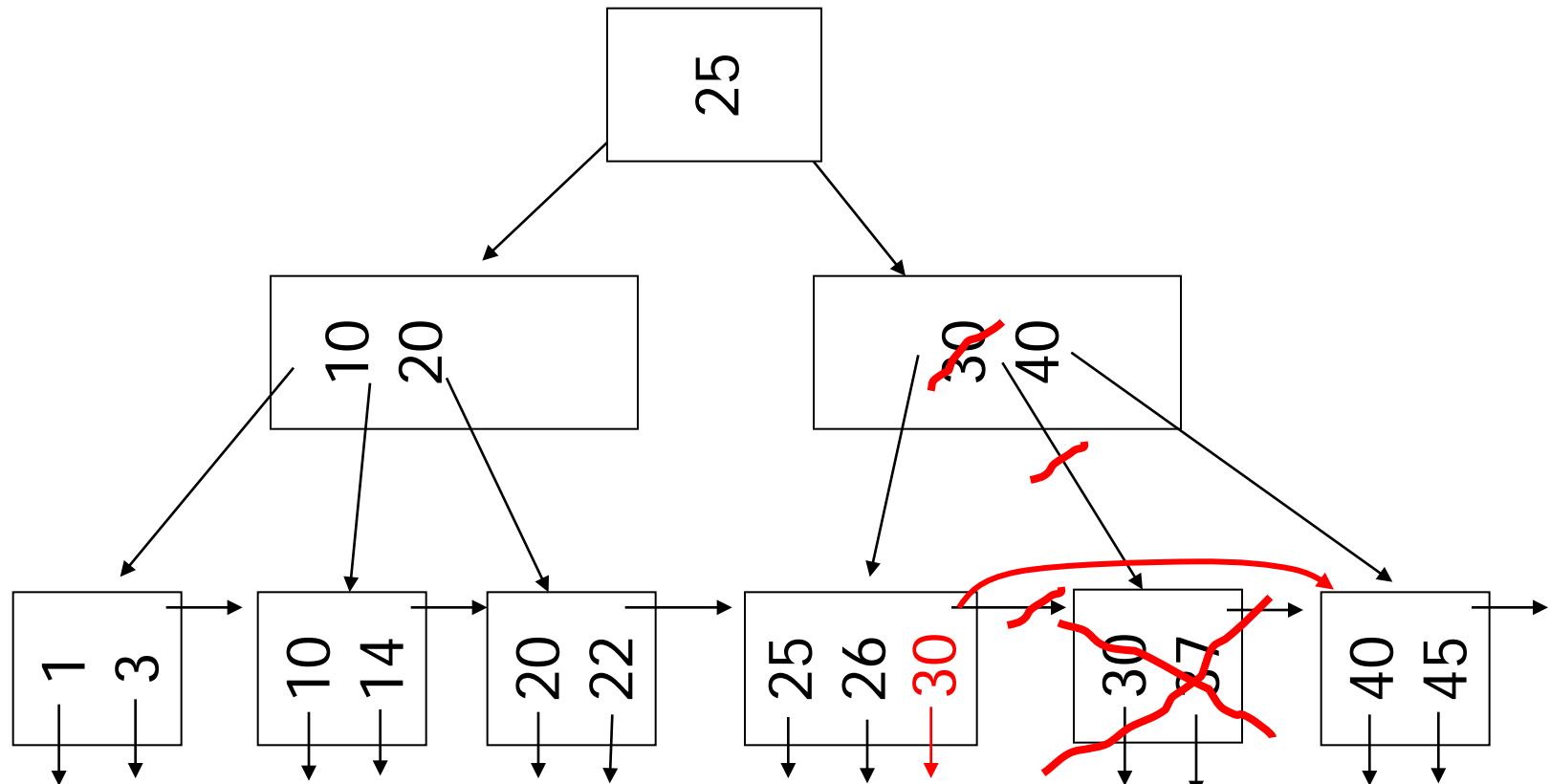
$n=4$
 $(n+1)/2$

(d) Non-leaf coalesce
– Delete 37



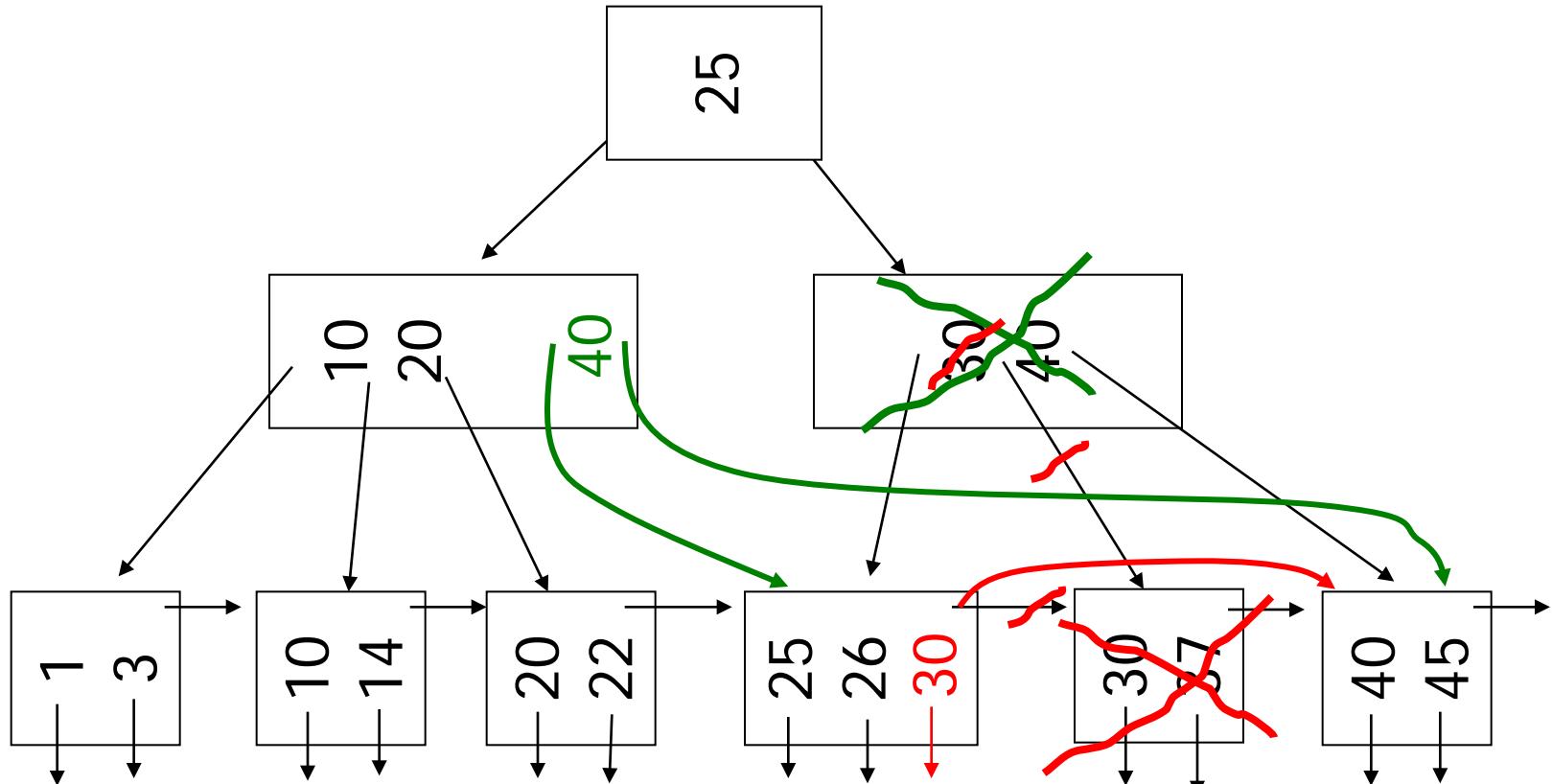
(d) Non-leaf coalesce
– Delete 37

$n=4$
 $(n+1)/2$



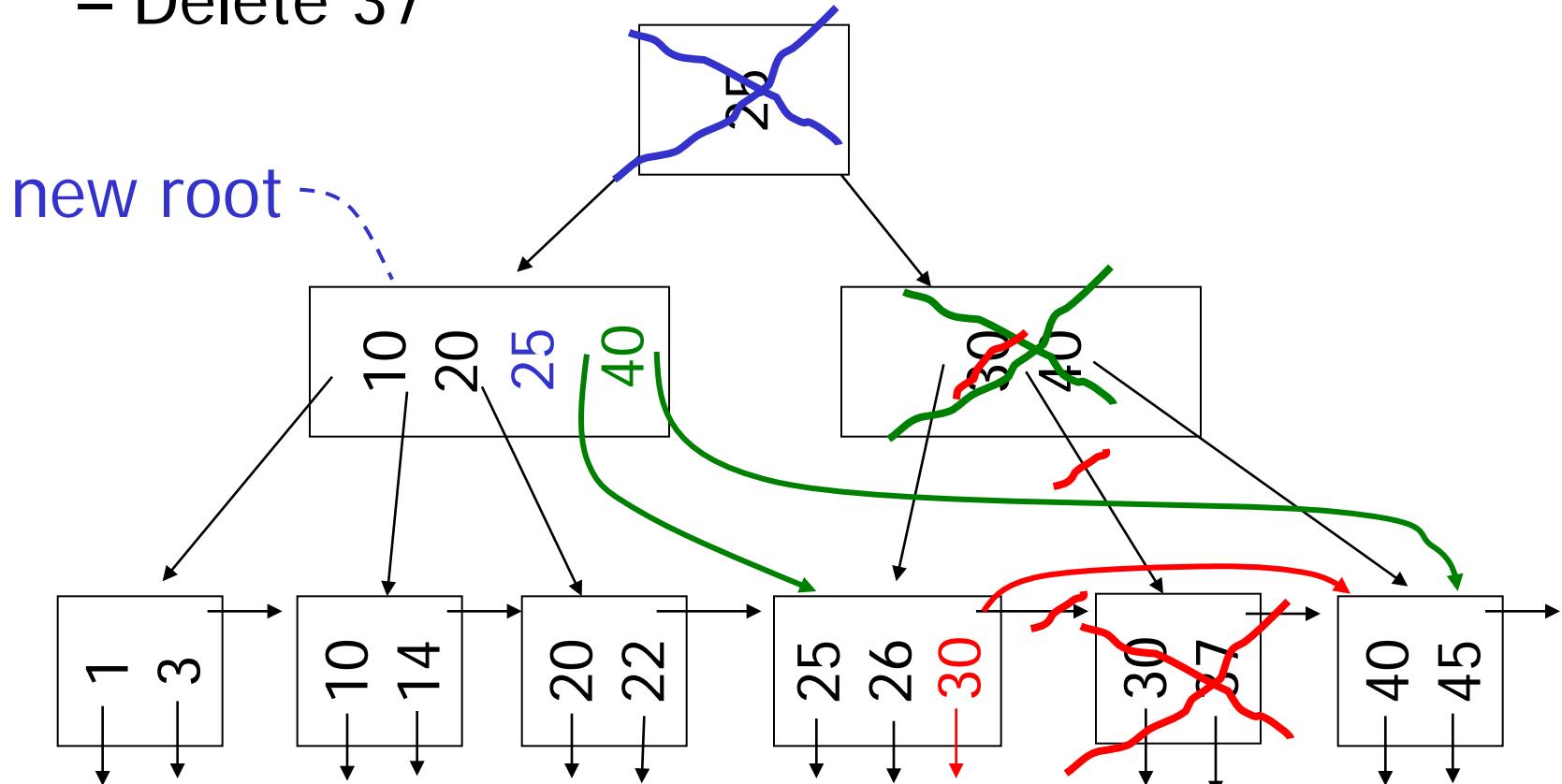
$n=4$
 $(n+1)/2$

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– Delete 37



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 $(n+1)/2$

(d) Non-leaf coalesce
– Delete 37





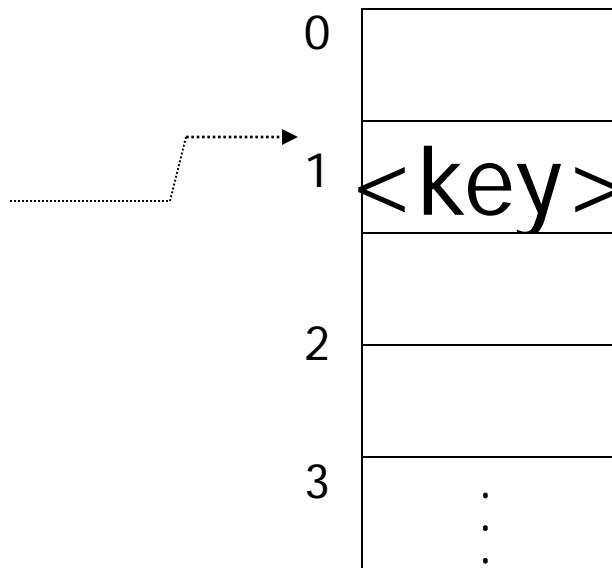
Topics

- Conventional Indexes
 - B-trees
- Hashing Schemes
- Multidimensional Indexes



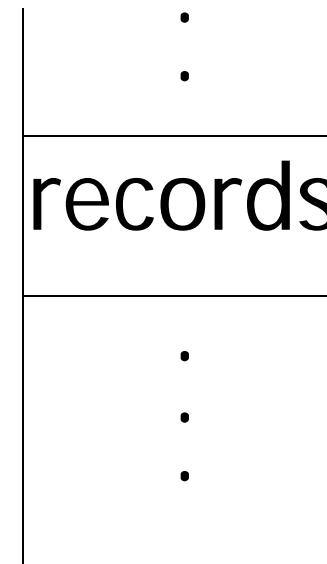
Hashing

$\text{key} \rightarrow h(\text{key})$



Two alternatives

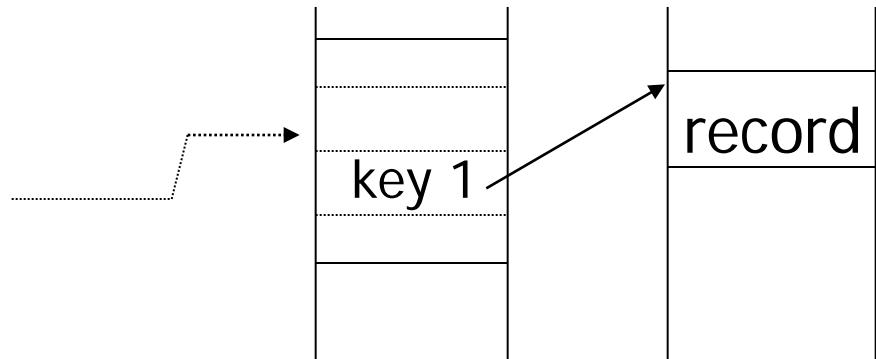
(1) key \rightarrow $h(\text{key})$



Bucket
(typically 1
disk block)

Two alternatives

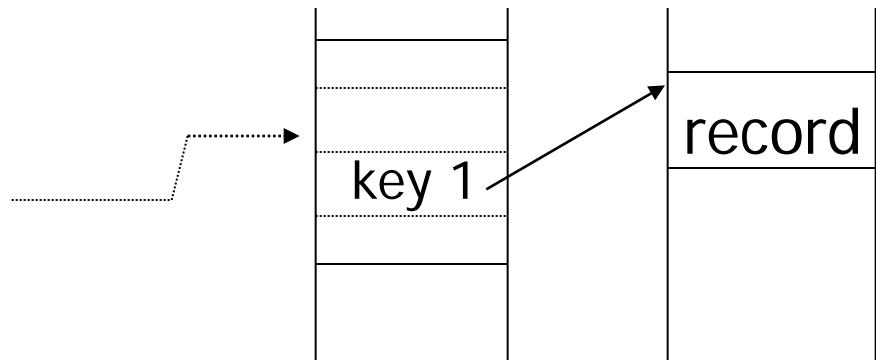
(2) key \rightarrow h(key)



Index

Two alternatives

(2) key → $h(\text{key})$



Index

- Alt (2) for “secondary” search key



Example hash function

- Key = ' $x_1 \ x_2 \dots \ x_n$ ' n byte character string
- Have b buckets
- h : add $x_1 + x_2 + \dots + x_n$
 - compute sum modulo b



- ☒ This may not be best function ...
- ☒ Read Knuth Vol. 3 if you really need to select a good function.



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Good hash
function:

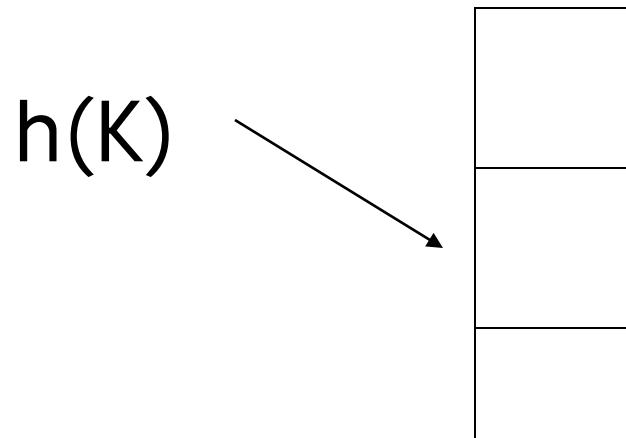
☞ Expected number of
hash-value/bucket is the
same for all buckets



Within a bucket:

- Do we keep keys sorted?
- Yes, if CPU time critical
 - & Inserts/Deletes not too frequent

Next: example to illustrate
inserts, overflows, deletes



EXAMPLE 2 records/bucket

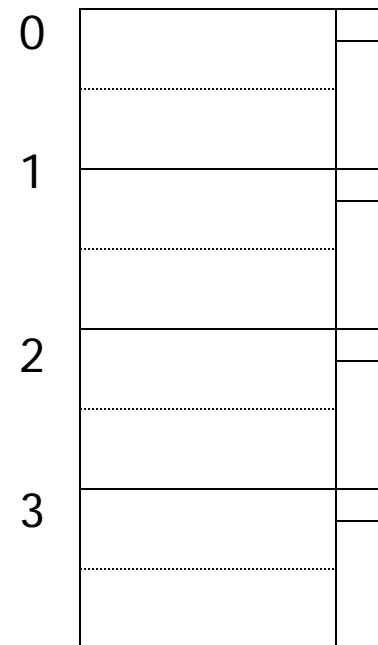
INSERT:

$h(a) = 1$

$h(b) = 2$

$h(c) = 1$

$h(d) = 0$



EXAMPLE 2 records/bucket

INSERT:

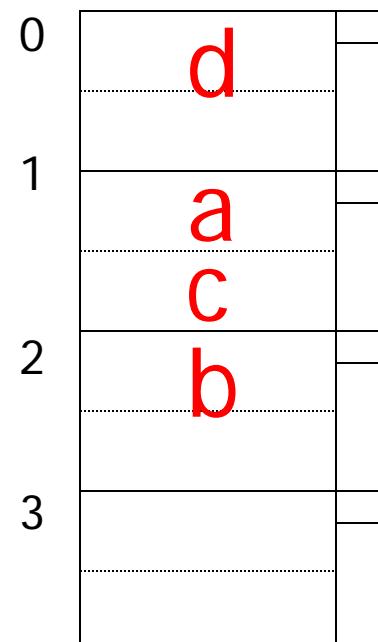
$$h(a) = 1$$

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$$h(d) = 0$$

$$h(e) = 1$$



EXAMPLE 2 records/bucket

INSERT:

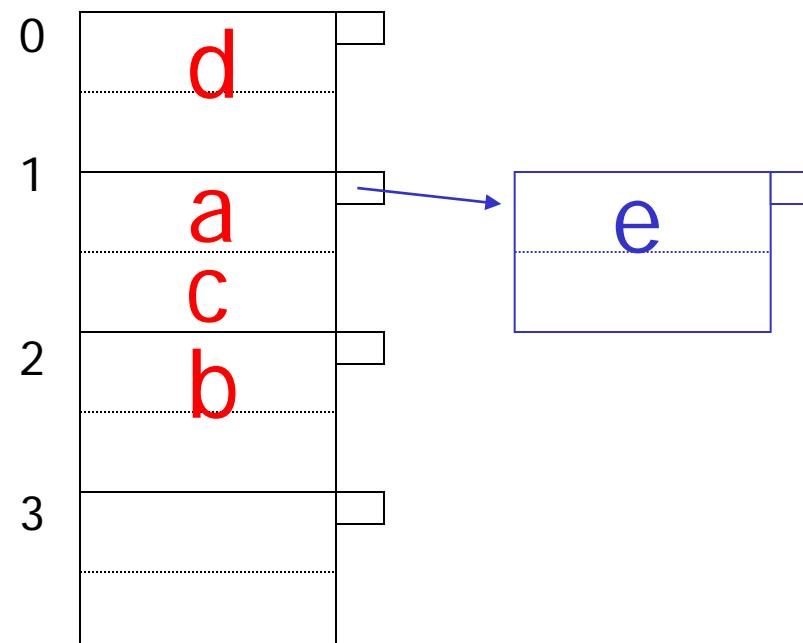
$$h(a) = 1$$

$$h(b) = 2$$

$$h(c) = 1$$

$$h(d) = 0$$

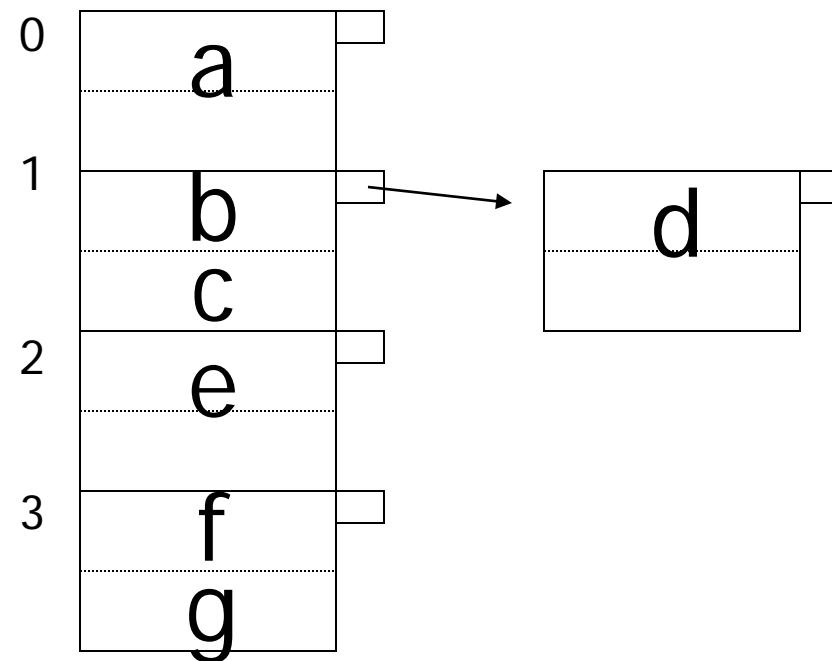
$$h(e) = 1$$



EXAMPLE: deletion

Delete:

e
f



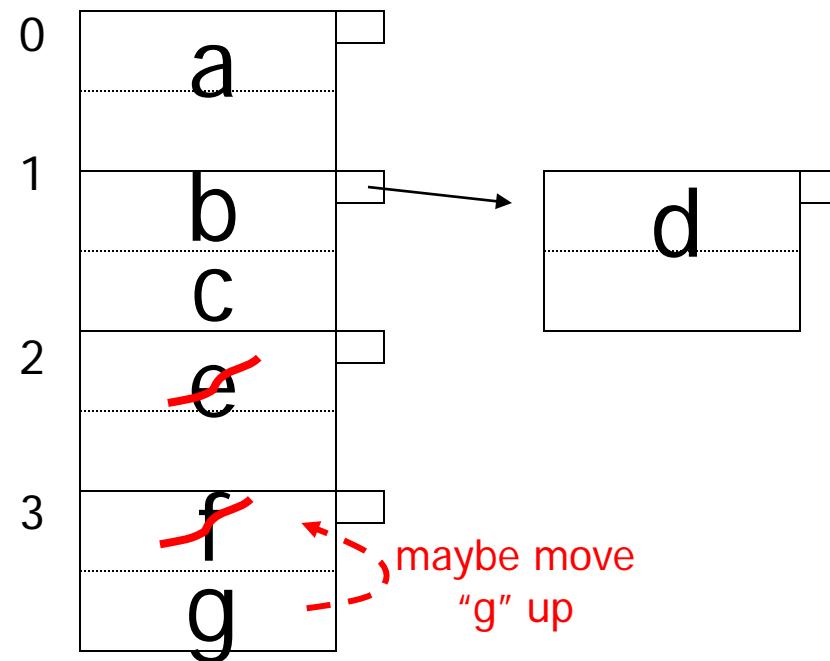
EXAMPLE: deletion

Delete:

e

f

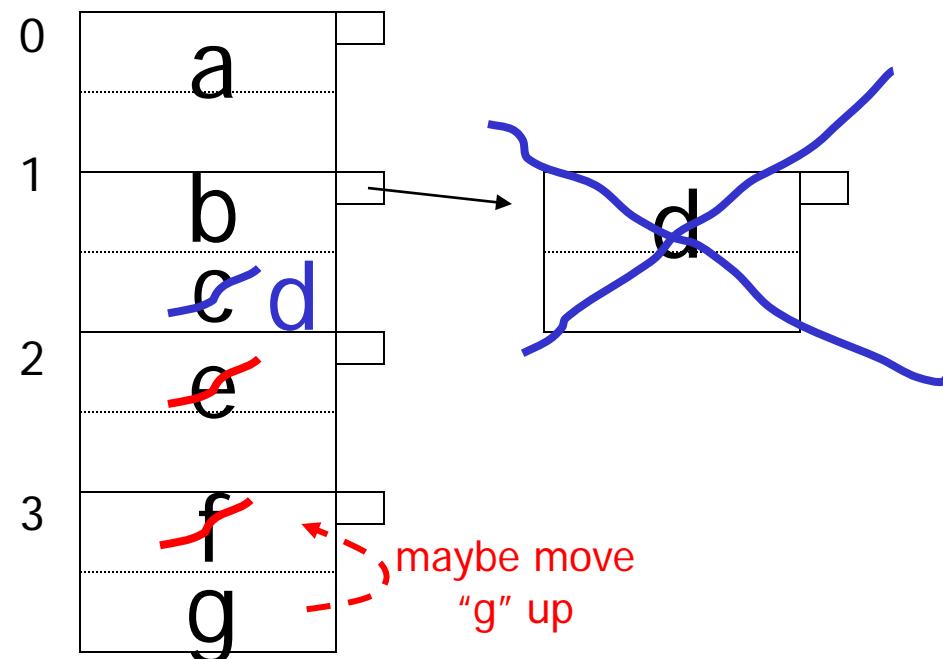
C



EXAMPLE: deletion

Delete:

e
f
c





Rule of thumb:

- Try to keep space utilization between 50% and 80%

$$\text{Utilization} = \frac{\text{\# keys used}}{\text{total \# keys that fit}}$$



Rule of thumb:

- Try to keep space utilization between 50% and 80%

$$\text{Utilization} = \frac{\text{\# keys used}}{\text{total \# keys that fit}}$$

- If < 50%, wasting space
- If > 80%, overflows significant
 - depends on how good hash function is & on # keys/bucket



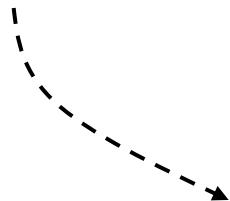
How do we cope with growth?

- Overflows and reorganizations
- Dynamic hashing



How do we cope with growth?

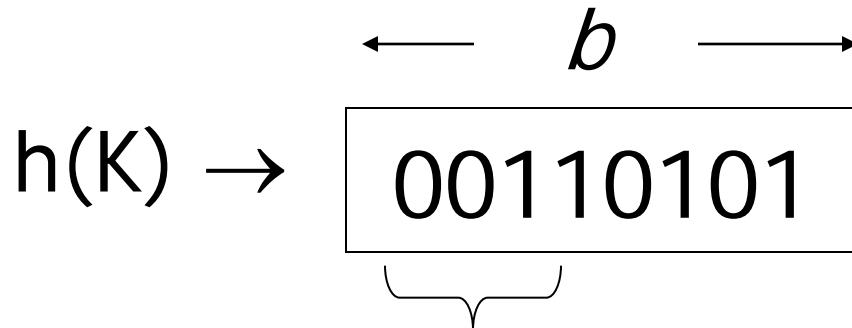
- Overflows and reorganizations
- Dynamic hashing



- Extensible
- Linear

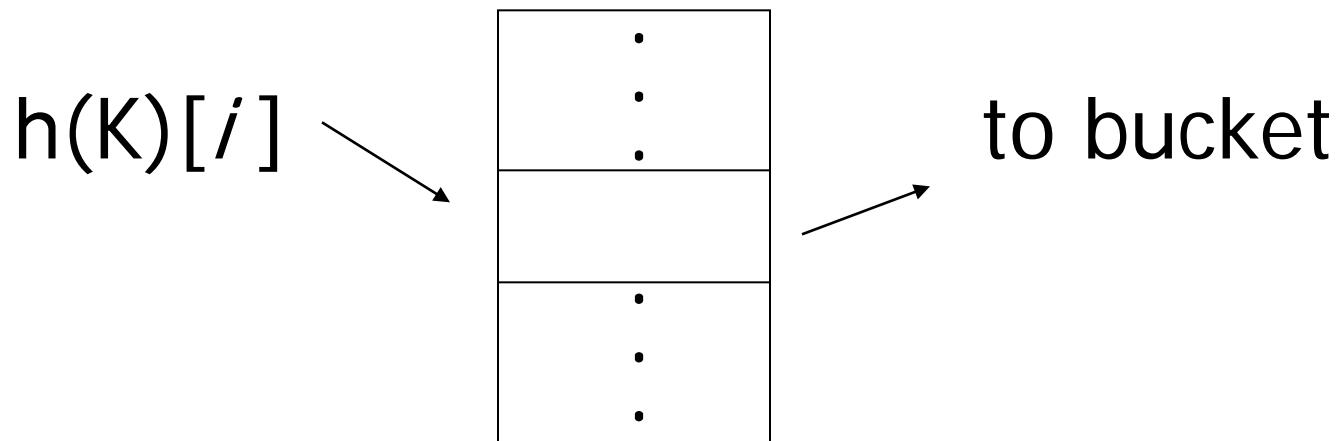
Extensible hashing: two ideas

(a) Use i of b bits output by hash function

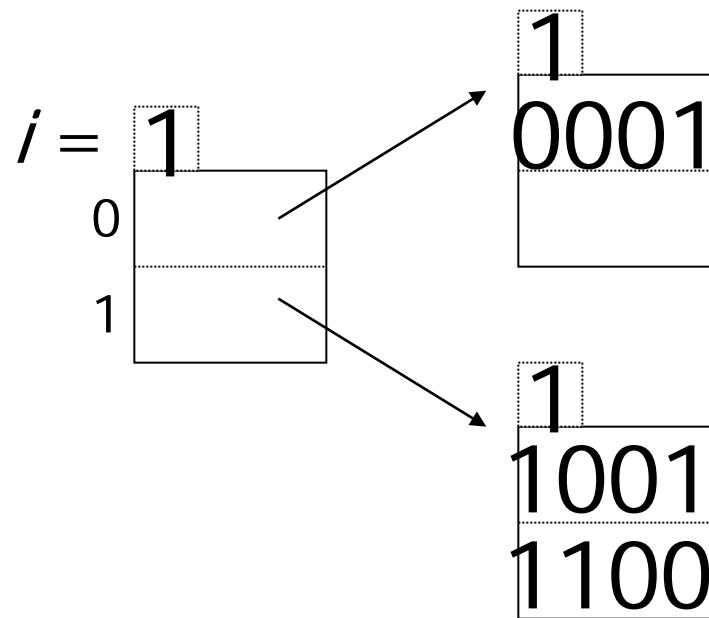


use $i \rightarrow$ grows over time....

(b) Use directory

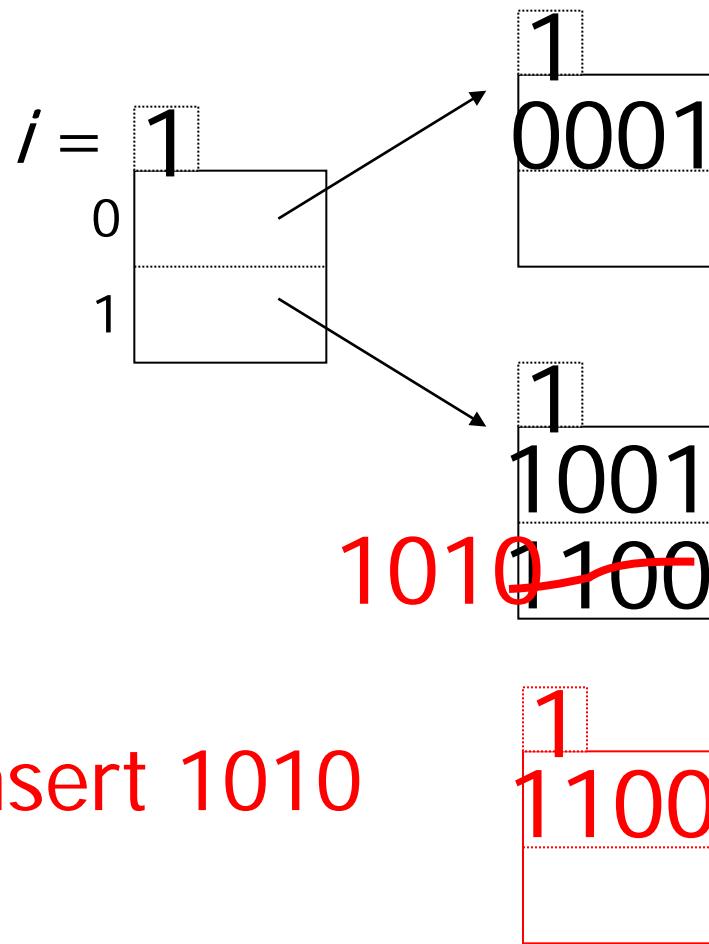


Example: $h(k)$ is 4 bits; 2 keys/bucket

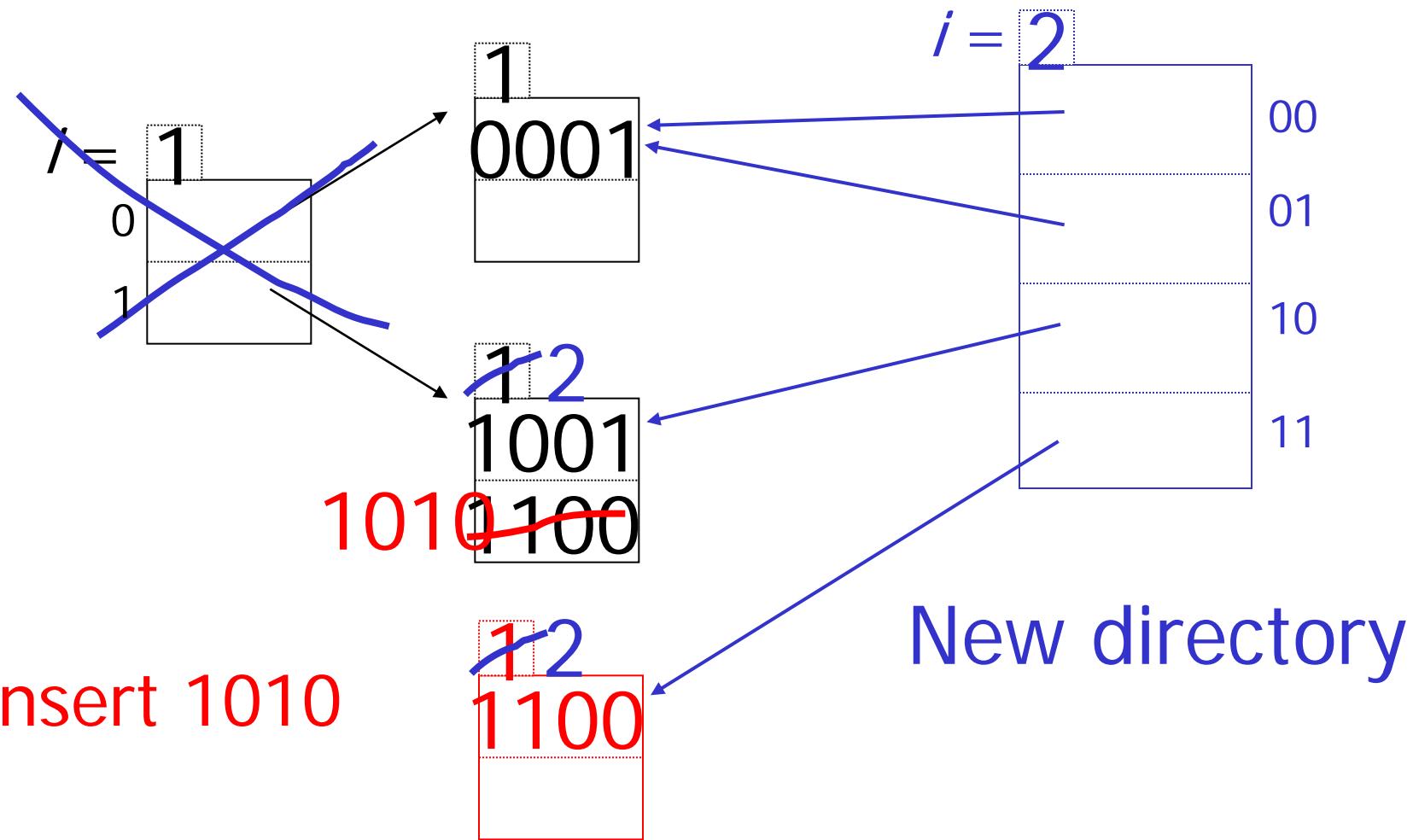


Insert 1010

Example: $h(k)$ is 4 bits; 2 keys/bucket



Example: $h(k)$ is 4 bits; 2 keys/bucket



Insert 1010

New directory

Example continued

$i = 2$

00

01

10

11

1

0001

2

1001

1010

2

1100

Insert:

0111

0000

Example continued

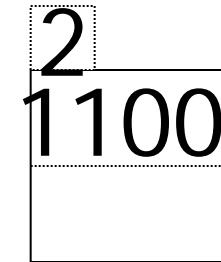
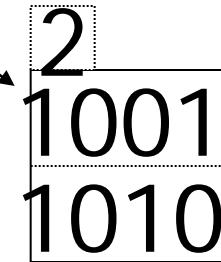
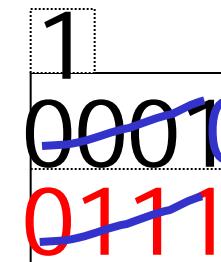
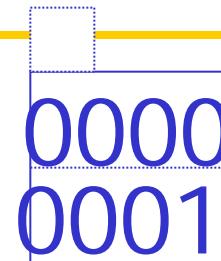
$i = 2$

00

01

10

11



Insert:

0111

0000

Example continued

$i = 2$

00

01

10

11

2	0000
	0001

12	0001
	0111

2	1001
	1010

2	1100

Insert:

0111

0000

Example continued

$i = 2$

00
01
10
11

0000₂
0001₂

0111₂

1001₂
1010₂

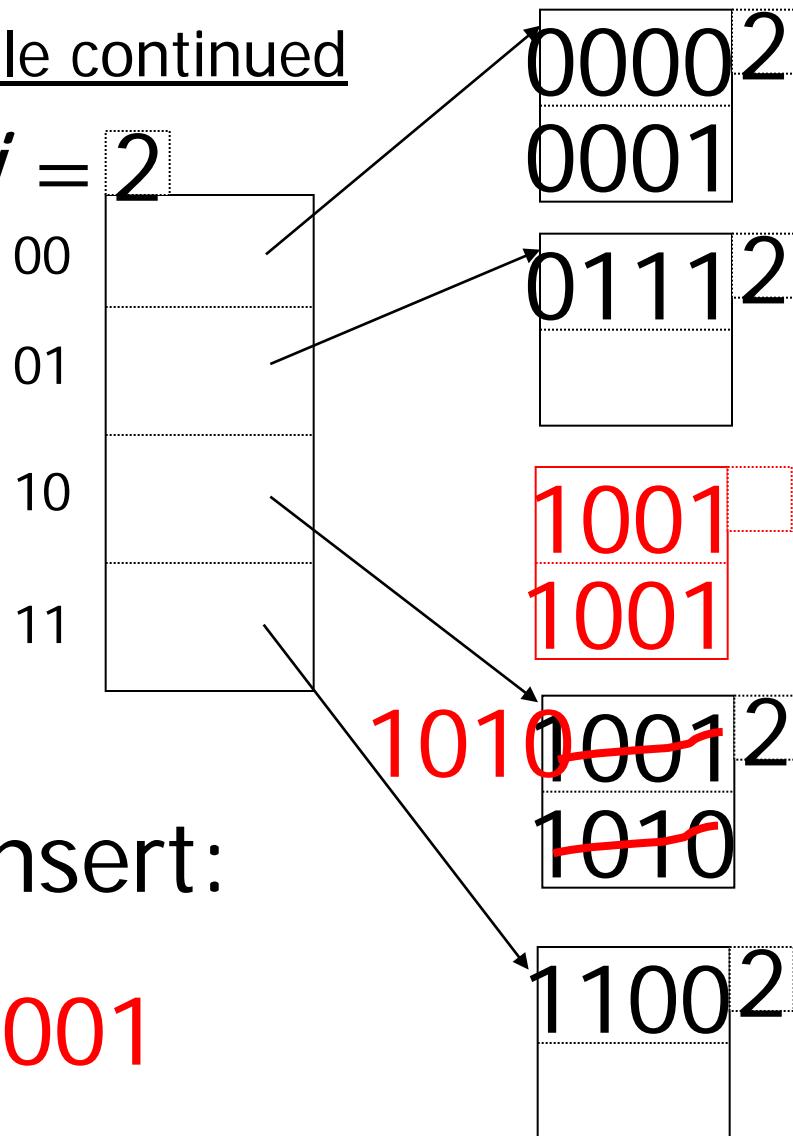
1100₂

Insert:

1001

Example continued

$$i = 2$$



Example continued

$i = 2$

00

01

10

11

00002
0001

01112

10013
1001

101000123
1010

11002

$i = 3$

000

001

010

011

100

101

110

111

Insert:

1001



Extensible hashing: deletion

- No merging of blocks
- Merge blocks
and cut directory if possible
(Reverse insert procedure)



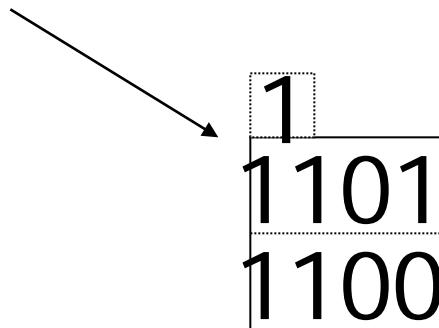
Deletion example:

- Run thru insert example in reverse!

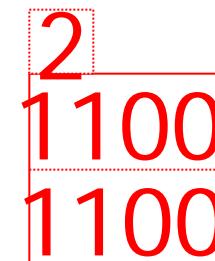
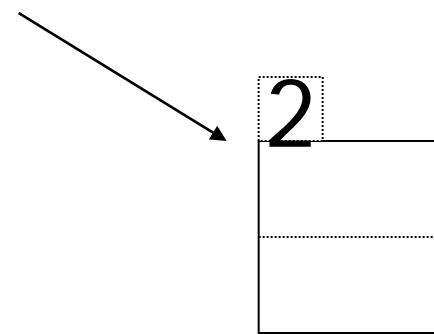
Note: Still need overflow chains

- Example: many records with duplicate keys

insert 1100

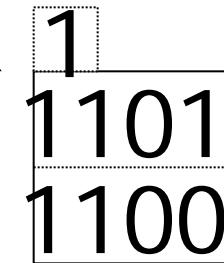


if we split:

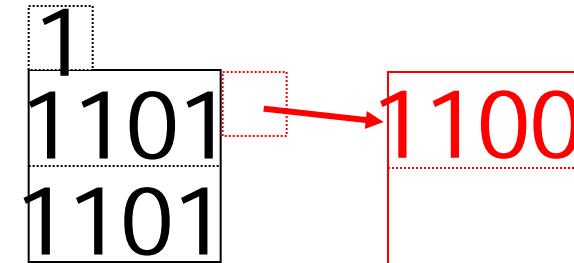


Solution: overflow chains

insert 1100



add overflow block:





Summary Extensible hashing

- ⊕ Can handle growing files
 - with less wasted space
 - with no full reorganizations



Summary Extensible hashing

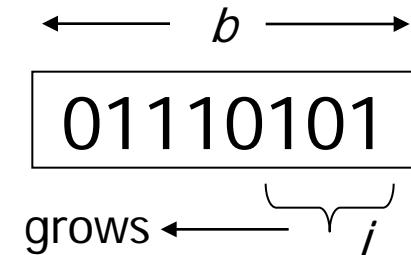
- ⊕ Can handle growing files
 - with less wasted space
 - with no full reorganizations
- ⊖ Indirection
 - (Not bad if directory in memory)
- ⊖ Directory doubles in size
 - (Now it fits, now it does not)

Linear hashing

- Another dynamic hashing scheme

Two ideas:

- (a) Use i low order bits of hash

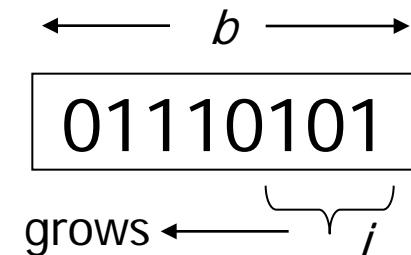


Linear hashing

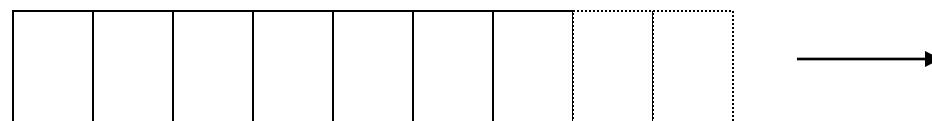
- Another dynamic hashing scheme

Two ideas:

- (a) Use i low order bits of hash



- (b) File grows linearly

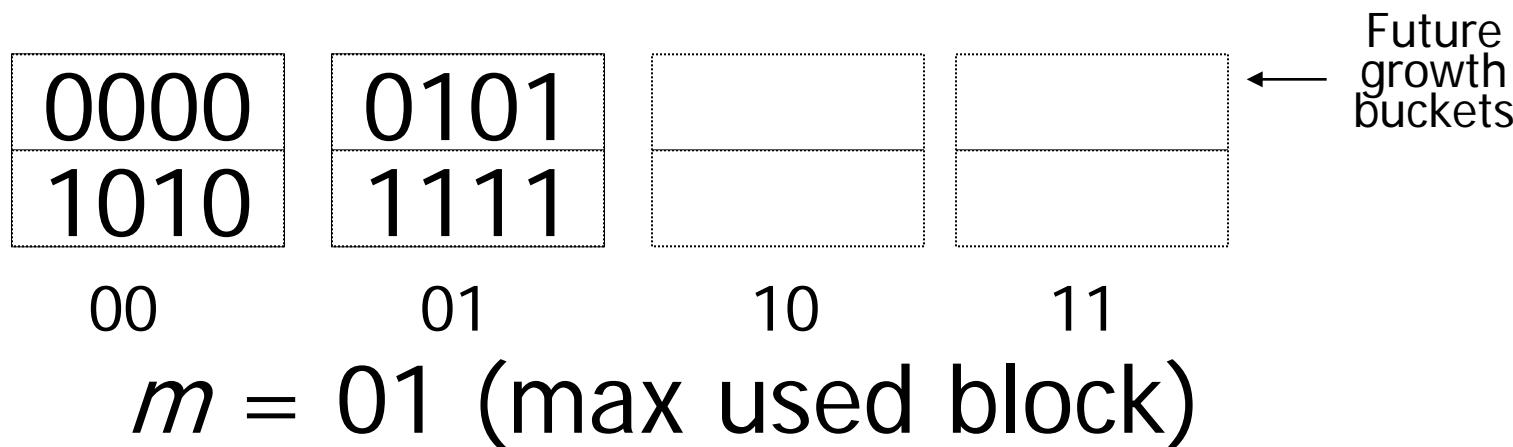




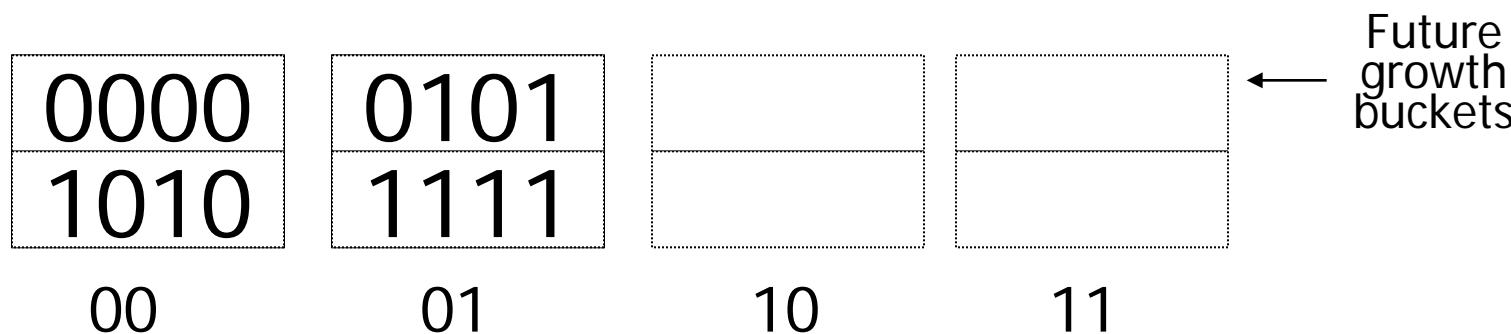
✉ When do we expand file?

- Keep track of: $\frac{\text{# used slots}}{\text{total # of slots}} = U$
- After every insertion, check if $U >$ threshold then increase buckets by 1
- If you run out of bits, add 1 more
 - 00 becomes 000

Example $b=4$ bits, $i=2$, 2 keys/bucket



Example $b=4$ bits, $i = 2$, 2 keys/bucket



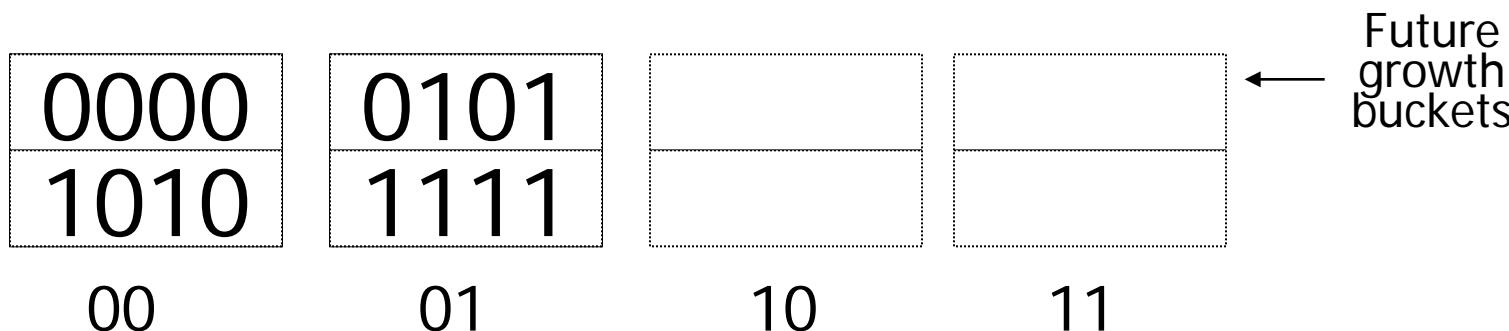
$m = 01$ (max used block)

Rule If $h(k)[i] \leq m$, then

look at bucket $h(k)[i]$
else, look at bucket $h(k)[i] - 2^{i-1}$

Example $b=4$ bits, $i=2$, 2 keys/bucket

- insert 0101



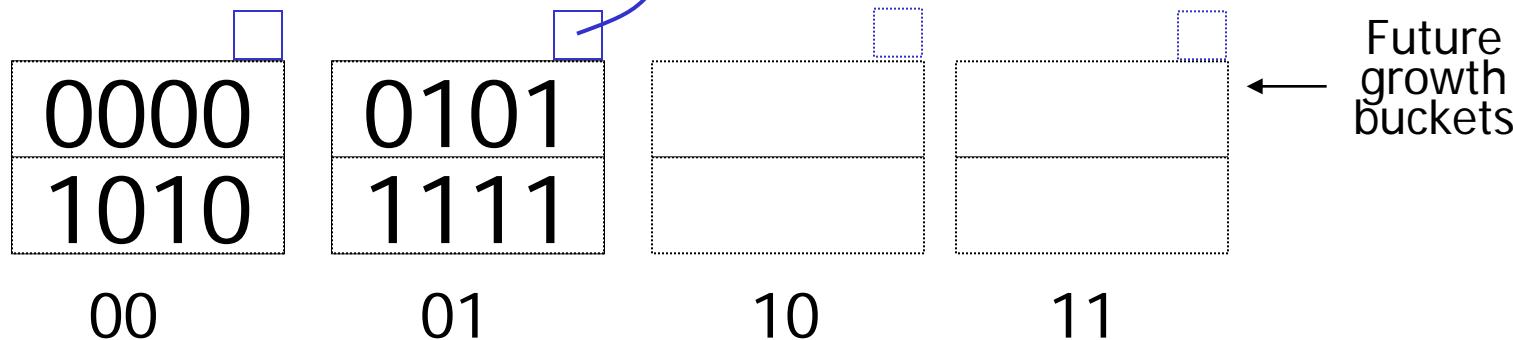
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Example $b=4$ bits, $i = 2$, 2 keys/bucket

- insert 0101
- can have overflow chains!

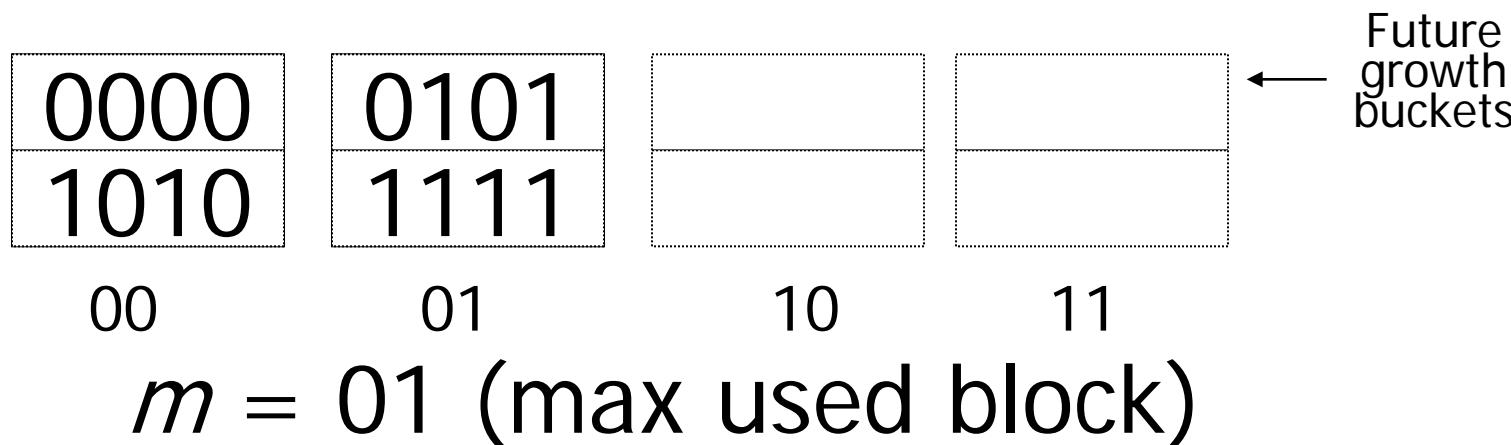


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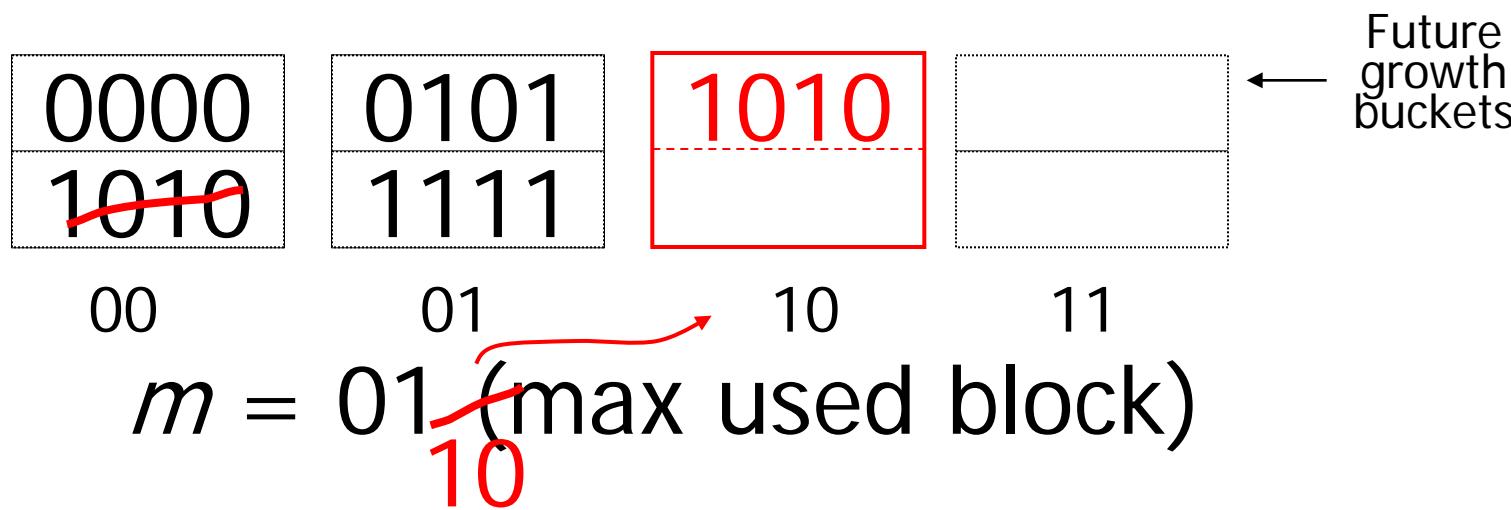
Example $b=4$ bits, $i=2$, 2 keys/bucket



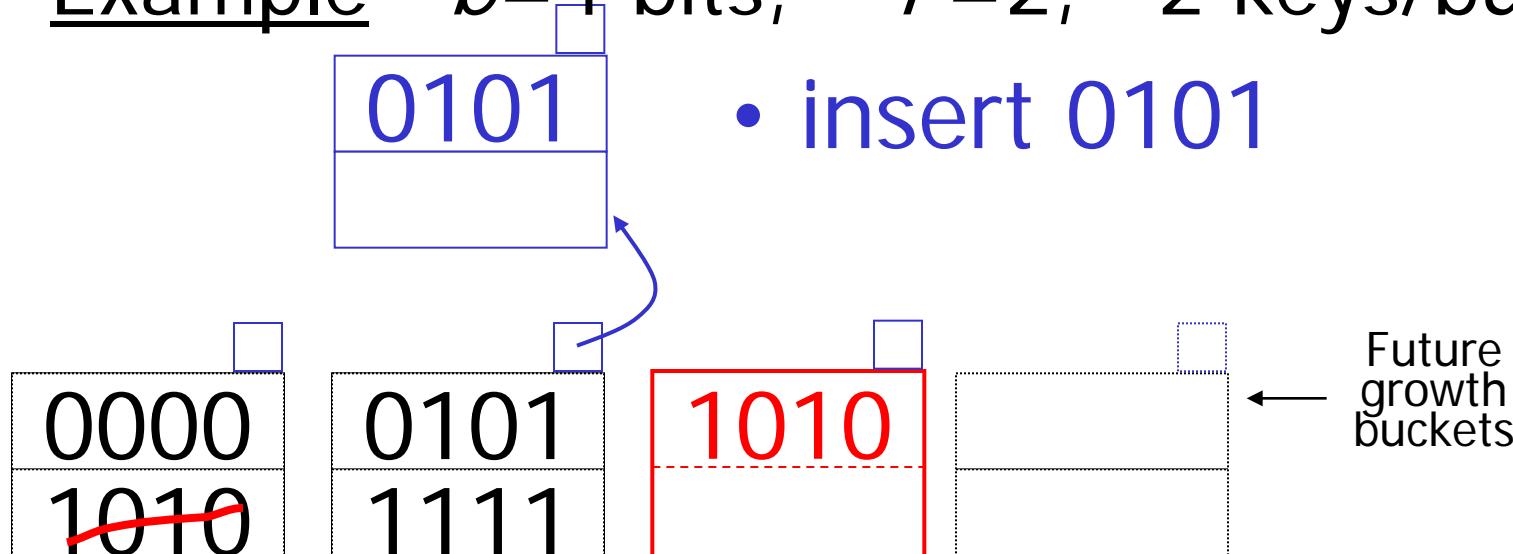
- After every insertion, check if $U >$ threshold then increase buckets by 1

$$U = \frac{\text{\# used slots}}{\text{total \# of slots}}$$

Example $b=4$ bits, $i=2$, 2 keys/bucket

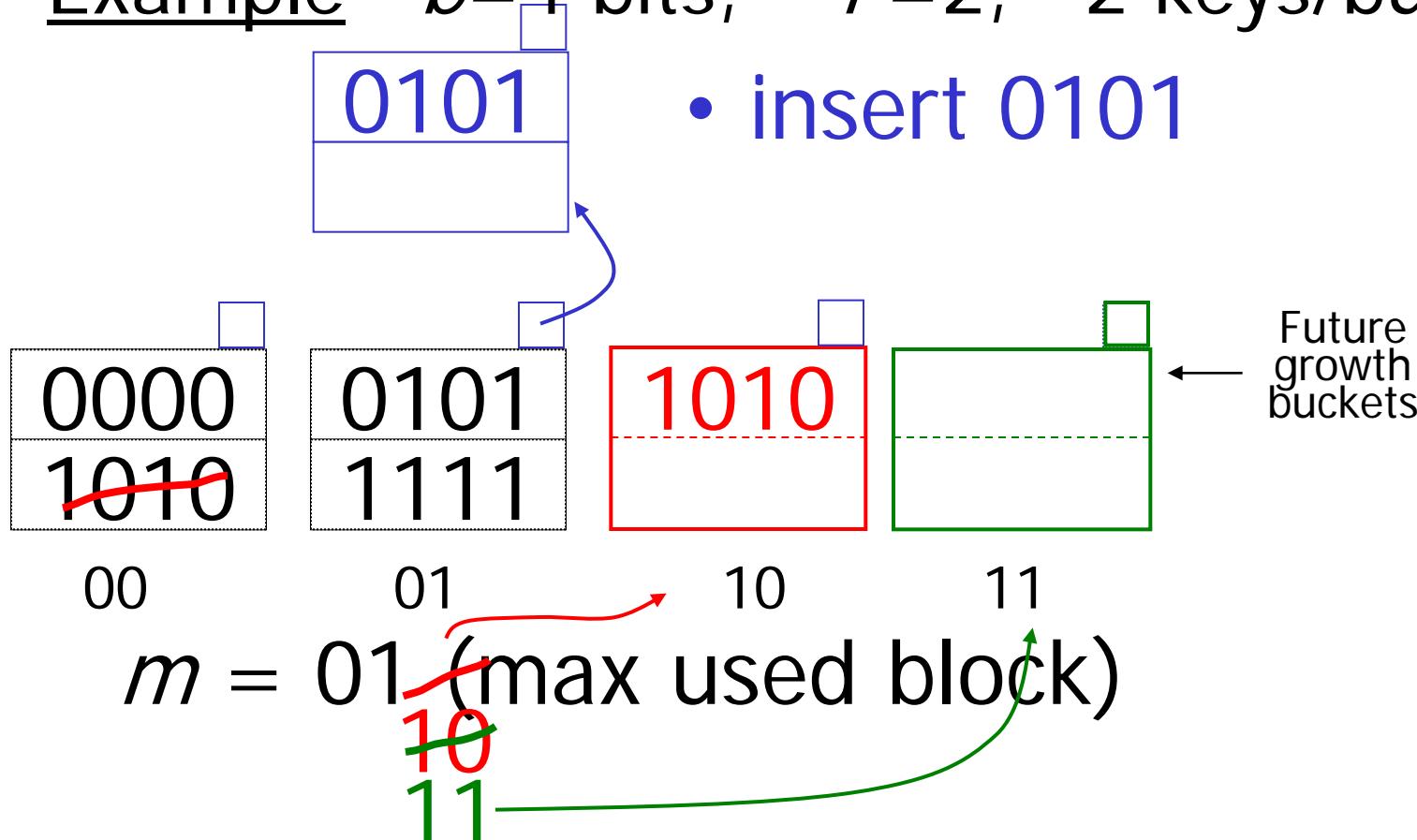


Example $b=4$ bits, $i=2$, 2 keys/bucket

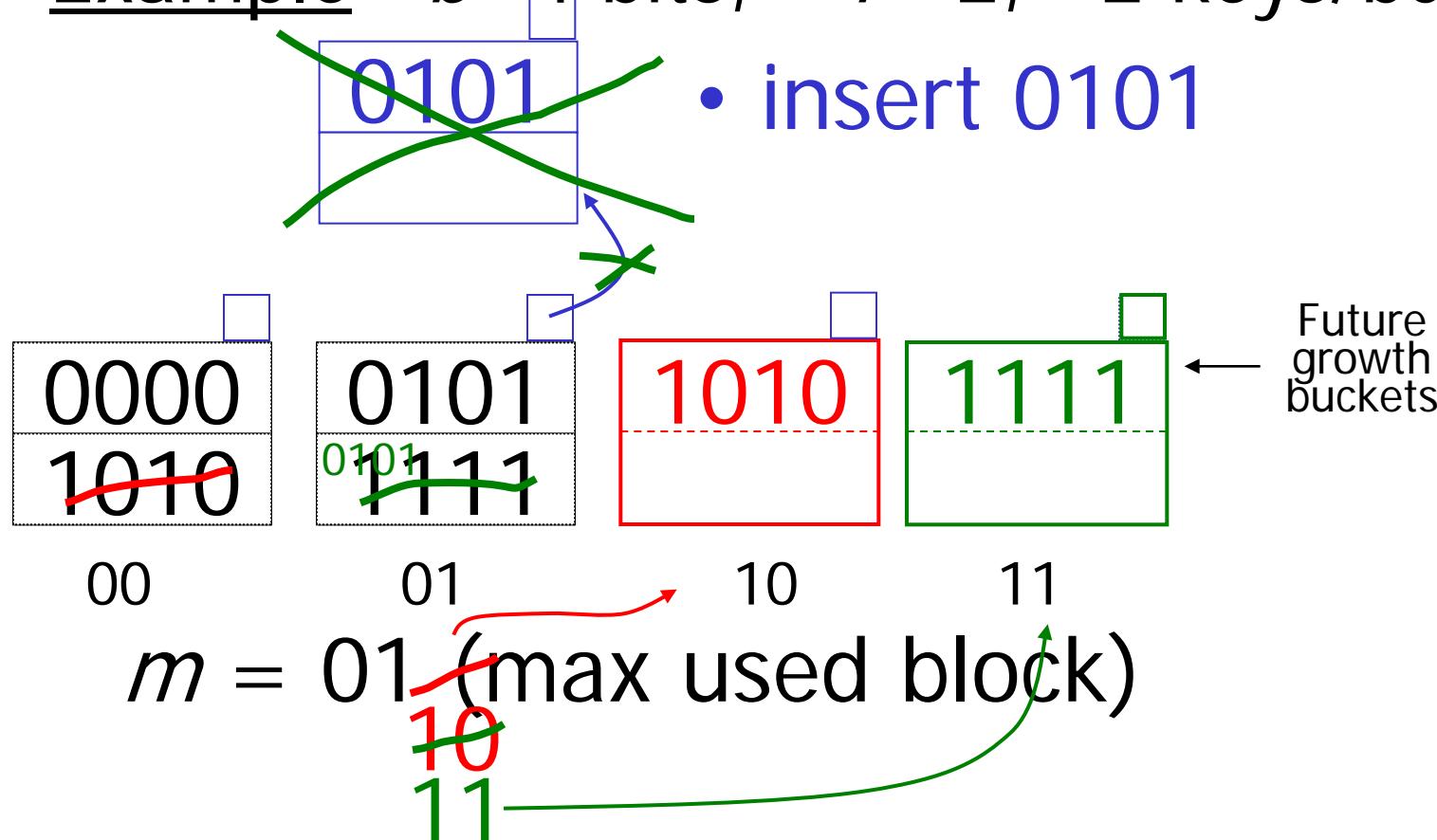


$m = 01$ (max used block)
10

Example $b=4$ bits, $i=2$, 2 keys/bucket



Example $b=4$ bits, $i=2$, 2 keys/bucket





Example Continued: How to grow beyond this?

$$i = 2$$

0000

00

0101

01

1010

10

1111

11

...

$$m = 11 \text{ (max used block)}$$

Example Continued: How to grow beyond this?

$$i = \cancel{23}$$

0000

0101
0101

1010

1111

0₀₀
100

0₀₁

101

0₁₀

0₁₁
110

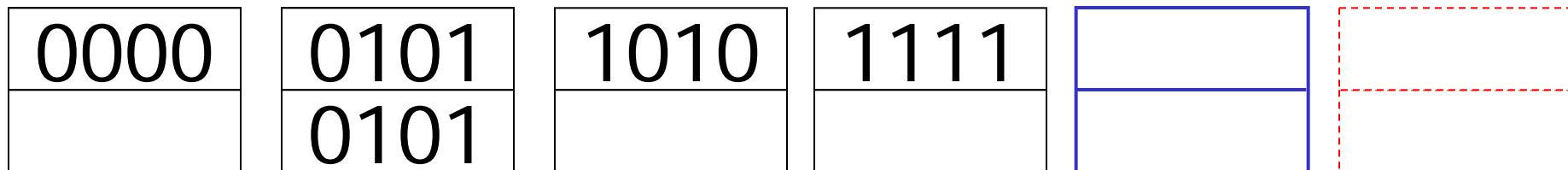
111

...

$m = 11$ (max used block)

Example Continued: How to grow beyond this?

$$i = \cancel{23}$$

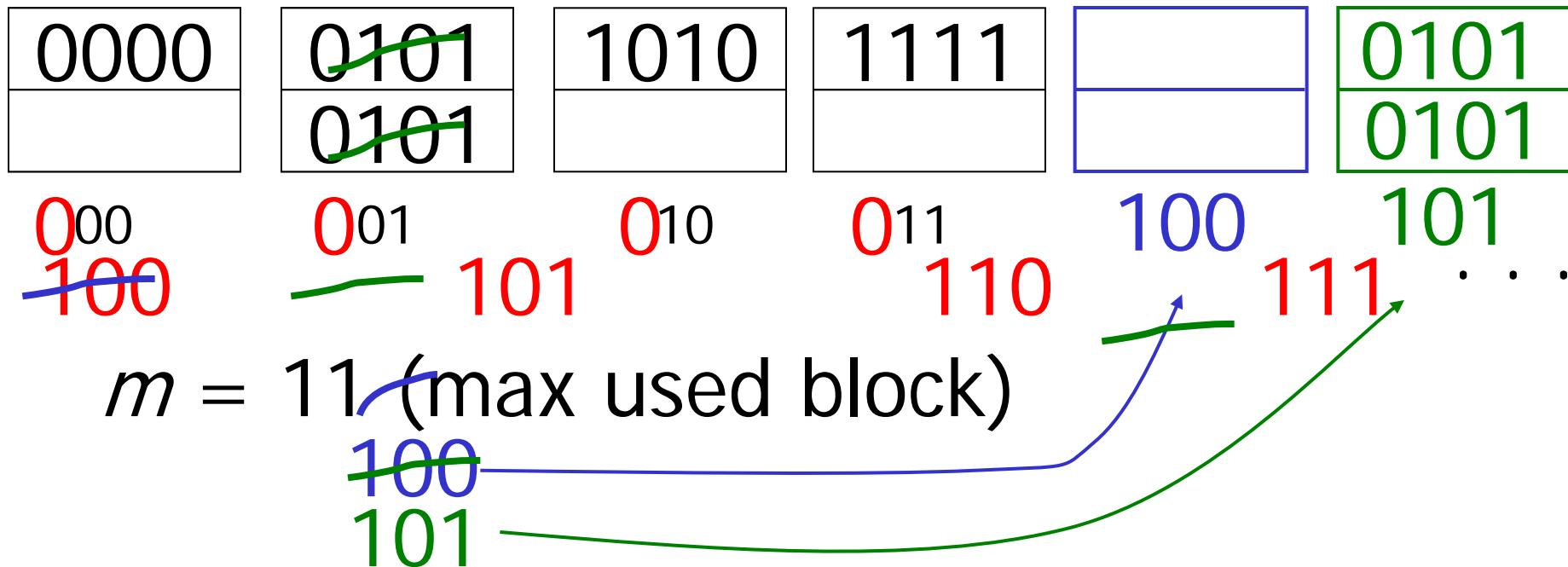


~~000
100~~ 001 101 010 011 110 100 111 ...

$m = 11$ (max used block)
~~100~~

Example Continued: How to grow beyond this?

$$i = \cancel{23}$$





Summary Linear Hashing

- ⊕ Can handle growing files
 - with less wasted space
 - with no full reorganizations
- ⊕ No indirection like extensible hashing
- ⊖ Can still have overflow chains